22. Phase-Locked Loop

A phase-locked loop (PLL) is an electronic circuit consisting of a variable frequency oscillator and a phase detector to generate an output signal whose phase is related to the phase of an input signal by feedback control technology. The oscillator generates a periodic signal and the phase detector adjusts the oscillator to keep the phases of input and output signals matched, i.e., to synchronize the input and output signals. The PLL are often used to demodulate a signal, generate a frequency at multiple of an input frequency or distribute precise clock pulses in digital logic circuits.

Let's consider two sinusoidal voltages, both the input and output voltages, respectively denoted as

$$g_i(t) = A\cos(\omega_c t + \phi_i(t)) \tag{1}$$

$$g_o(t) = B\sin(\omega_c t + \phi_o(t)) \tag{2}$$

whose product with a gain k is given by

$$v(t) = kg(t)h(t) = k\frac{AB}{2}\{\sin(2\omega_c t + \phi_i(t) + \phi_0(t)) + \sin(\phi_i(t) - \phi_0(t))\} \tag{3}$$

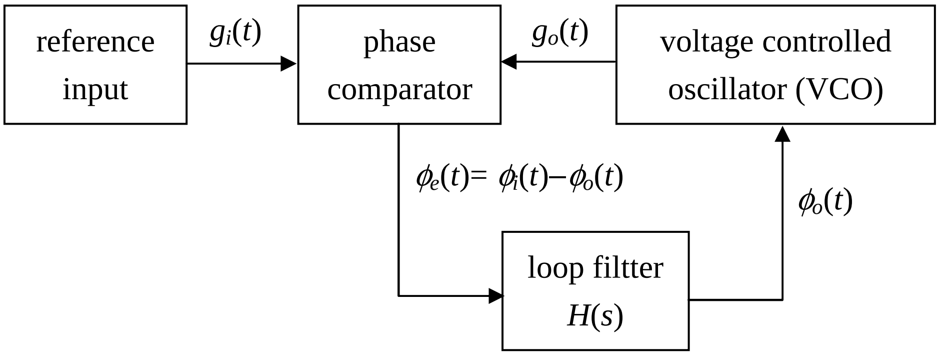
When $v(t)$ is passed through a low-pass filter and the difference $\phi_i(t) - \phi_0(t)$ is small, we approximately obtain an output as below

$$v_o(t) = k\frac{AB}{2}\sin(\phi_i(t) - \phi_o(t)) \approx k\frac{AB}{2}(\phi_i(t) - \phi_o(t)) \tag{4}$$

Obviously, the difference in phase is detected. Usually, we call the circuit to detect the phase difference is a phase comparator or a phase detector. We assume the result of the phase comparator is

$$\phi_e(t) = \phi_i(t) - \phi_o(t) \tag{5}$$

To reduce the phase difference $\phi_e(t)$, further design a filter $H(s)$ to generate the output $\phi_o(t)$ which can successfully track the input phase $\phi_e(t)$. A general structure of PLL is depicted in the figure consisting of the filter $H(s)$ which receives the signal $\phi_e(t)$ from the phase comparator and sends out the appropriate output phase $\phi_o(t)$ to the voltage controlled oscillator (VCO).



Based on the Laplace transform, the feedback control system can be described as the following convolution form

$$\Phi_o(s) = H(s)\Phi_e(s) \tag{6}$$

where $\Phi_e(s)$ and $\Phi_o(s)$ are the Laplace transform of $\phi_e(t)$ and $\phi_o(t)$. With proper design of the filter $H(s)$, the phase $\phi_o(t)$ of the output signal can be adjusted to approach the phase $\phi_i(t)$ of input signal as $t \to \infty$.