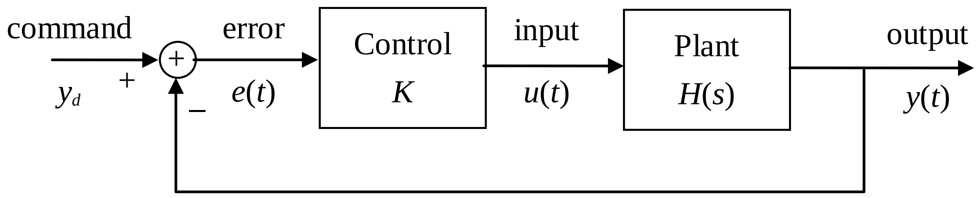
19. Unit Output Feedback Control

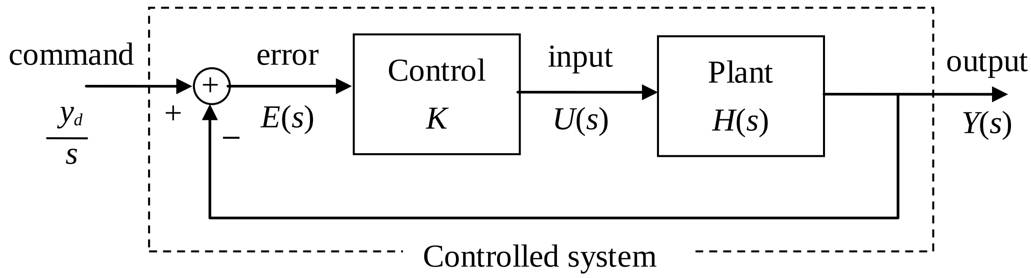
Here, we will introduce the simplest control scheme called the unit output feedback control and depicted in the following block diagram, where $H(s)$ is the transfer function of the plant to be controlled, K is the constant feedback control, $u(t)$ is the plant input, $y(t)$ is the plant output, y_d is the command or desired output, and $e(t)$ is error signal of $y(t)$ deviated from y_d.



For simplicity, let consider a second order plant as an example whose transfer function is given as below:

$$H(s) = \frac{cs + d}{s^2 + as + b} \tag{1}$$

With the Laplace transform, the total system can be redrawn as the following block diagram, where $U(s)$, $Y(s)$ and $E(s)$ are the Laplace transforms of input, output and error signal.



From the block diagram, the command $\frac{y_d}{s}$ is the input of the controlled system, the error is

$$E(s)=\frac{y_d}{s}-Y(s) \tag{2}$$

the input is

$$U(s)=KE(s) \tag{3}$$

and the output is

$$Y(s)=H(s)U(s) \tag{4}$$

From (2), (3) and (4), we obtain

$$Y(s)=H(s)U(s)=KH(s)E(s)=KH(s)\left(\frac{y_d}{s}-Y(s)\right) \tag{5}$$

and further rearranging it yields

$$Y(s)=\frac{KH(s)}{1+KH(s)}\left(\frac{y_d}{s}\right) \tag{6}$$

From (1), we have

$$Y(s)=\frac{Kcs+Kd}{s^2+(a+Kc)s+b+Kd}\left(\frac{y_d}{s}\right) \tag{7}$$

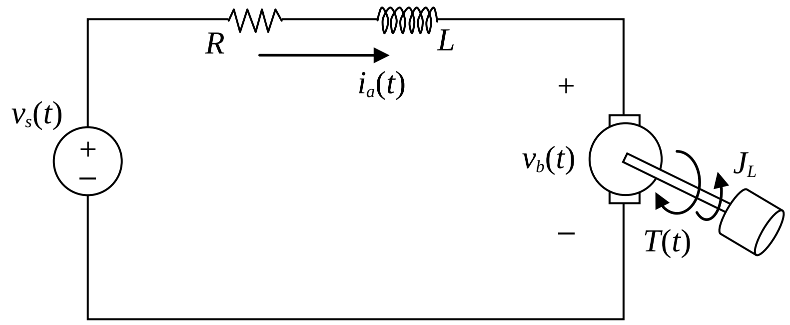
From the final value theorem, if the two roots of $s^2+(a+Kc)s+b+Kd=0$ are in the complex left-half plane, then for $t\to\infty$ the output will be

$$\begin{aligned}\lim_{t\to\infty} y(t)&=\lim_{s\to 0} sY(s)\\ &=\left.\frac{(Kcs+Kd)y_d}{s^2+(a+Kc)s+b+Kd}\right|_{s=0}=\frac{Kd}{b+Kd}y_d\end{aligned} \tag{8}$$

It is obvious that if b=0, then the output can be successfully controlled to the desired output, i.e., $\lim_{t\to\infty} y(t)=y_d$, otherwise there exists a steady-state error.

On the other hand, if c=0 and a<0, then the two roots can not be adjusted to the complex left-half plane. In other words, it is not controllable. Moreover, if cd<0, c and d not with the same numeric sign, it will be more difficult to design the control gain K. A plant with cd<0 is called a nonminimum plant.

Example: Motor vvelocity control



The structure of a DC motor is shown above, including the armature resistance R and winding leakage inductance L. The dynamic equation is

$$Ri_a(t) + L\frac{di_a(t)}{dt} + k\omega(t) = v_s(t) \tag{9}$$

$$J\frac{d\omega(t)}{dt}(t) + B\omega(t) - ki_a(t) = 0 \tag{10}$$

where $i_a(t)$ is the armature current, $v_s(t)$ is the voltage source and $\omega(t)$ is the angular velocity. From (9) and (10), we have

$$J\ddot{\omega}(t) + \left(B + \frac{RJ}{L}\right)\dot{\omega}(t) + \left(\frac{BR}{L} + \frac{k^2}{L}\right)\omega(t) = \frac{k}{L}v_s(t) \tag{11}$$

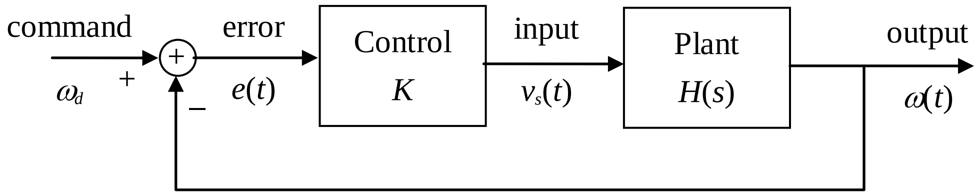
which can be rewritten as

$$\ddot{\omega}(t) + a\dot{\omega}(t) + b\omega(t) = dv_s(t) \tag{12}$$

with $a = \frac{B}{J} + \frac{R}{L}, b = \frac{BR}{JL} + \frac{k^2}{JL}$ and $d = \frac{k}{JL}$. The transfer function is

$$H(s) = \frac{d}{s^2 + as + b} \tag{13}$$

In order to control the motor to move in a desired velocity ω_d, we use the unit output feedback control shown as below:



Then, the resulted output is

$$\Omega(s) = \frac{Kd}{s^2 + as + b + Kd}\left(\frac{\omega_d}{s}\right) \tag{14}$$

where the two roots of $s^2 + as + b + Kd = 0$ are in the complex left-half plane. Hence, as $t \to \infty$ the output will be

$$\begin{aligned} \lim_{t\to\infty} y(t) &= \lim_{s\to 0} sY(s) \\ &= \left.\frac{Kd\omega_d}{s^2 + as + b + Kd}\right|_{s=0} = \frac{Kd}{b + Kd}\omega_d \end{aligned} \tag{15}$$

and a steady-state error exists.