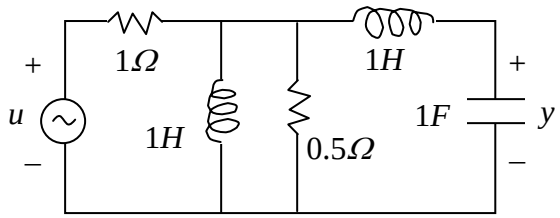
Signals And Systems Exam#2

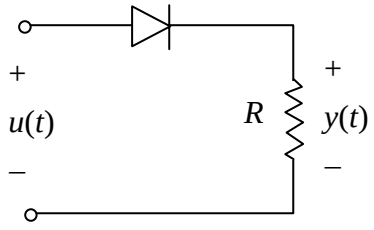
1. (8%)

Find the transfer function from *u* to *y* of the following circuit:



2. (18%)

An ideal half-wave rectifier is given below:



(A) What is the fundamental period of $y(t)$ if $u(t) = sin(t)$? (2%)

(B) What is the average power of $y(t)$? (3%)

(C) Find $y(t)$ in complex Fourier series. (8%)

(D) Find the Fourier transform of $y(t)$. (5%)

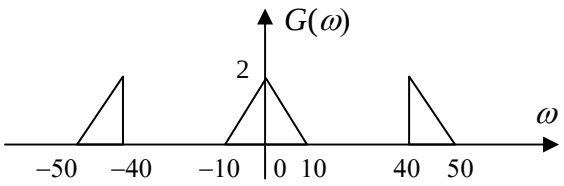
3. (10%)

Plot the magnitude spectrum and phase spectrum of the following signals:

(A) $x(t) = 2 - sin(2t) + 3sin(3t) + cos(2t)$ (4%)

(B) $y(t) = \begin{cases} e^{-t} & t \geq 0 \\ e^{2t} & t < 0 \end{cases}$ (6%)

4. (12%)



Consider the frequency spectrum of $g(t)$:

(A) Plot the frequency spectrum of $g(t)cos20t$.

(B) Let the spectrum of $h(t)$ be

$$H(\omega) = \begin{cases} 1 & |\omega| \leq 30 \\ 0 & |\omega| > 30 \end{cases}$$

Compute the total energy of

$$\left(g(t) \cdot cos^2(200t)\right) * h(t)$$

5. (14%)

A transfer function of an LTI system is given as

$$H(s) = \frac{2s^3 + 4s^2 + 3s}{s^4 + 4s^3 + 7s^2 + 6s + 2}$$

in which the input and output are denoted as $u(t)$ and $y(t)$.

(A) Find the differential equation of the system. (4%)

(B) Determine its impulse response. (10%)

6. (20%)

Consider an initially relaxed LTI system. When the system is excited by $u(t)=sin(t)$ for $t \geq 0$, its output is measured as $y(t) = 0.5e^{-t} - e^{-2t} + 0.5e^{-3t}$. Clearly, the system is not excited by $sin(t)$.

(A) What is its impulse response? (6%)

(B) What is the output if $u(t)=cos(t)$ for $t \geq 0$ and the initial conditions are $y(0) = 1$ and $\dot{y}(0) = \ddot{y}(0) = 0$. (8%)

(C) Can you find a set of initial conditions $y(0)$, $\dot{y}(0)$, and $\ddot{y}(0)$, such that the free response without input is the same as $y(t) = 0.5e^{-t} - e^{-2t} + 0.5e^{-3t}$? (6%)

7. (18%)

Consider the following Fourier transform pairs:

$f_1(t) \leftrightarrow G(\omega)$ (8%)

$f_2(t) \leftrightarrow j\omega \cdot G(\omega)$ (4%)

$f_3(t) \leftrightarrow G(\omega - 1)$ (3%)

$f_4(t) \leftrightarrow e^{j\omega} G(\omega)$ (3%)

where $G(\omega)$ is given as

$$G(\omega) = \begin{cases} 2 & |\omega| \leq 3 \\ 0 & |\omega| > 3 \end{cases}$$

Please determine the signals $f_i(t)$., i=1,2,3,4.