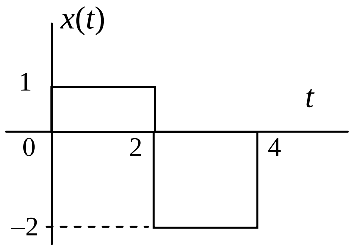
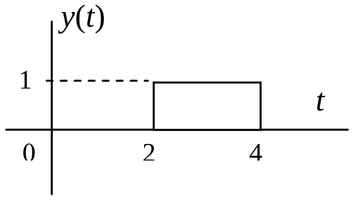
Signals And Systems Exam#1

1. Given *x*(*t*) and *y*(*t*) below:



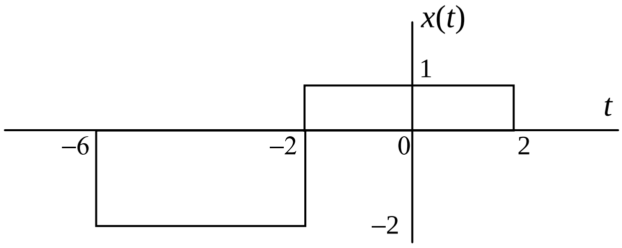


(A) Give the expression of $x(t)$ in terms of step functions. (3%)

Ans $x(t)=q(t)-3q(t-2)+2q(t-4)$

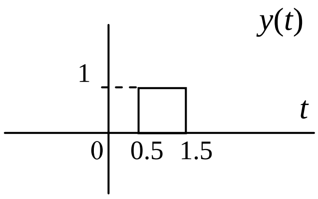
(B) Plot $x(1–0.5t)$. (3%)

Ans $x(t)\rightarrow g(t)=x(-t)\rightarrow h(t)=g(0.5t)=x(-0.5t)\rightarrow p(t)=h(t-2)=x(1-0.5t)$



(C) Plot $y(2t+1)$. (3%)

Ans $y(t)\rightarrow g(t)=y(2t)\rightarrow h(t)=g(t+0.5)=y(2t+1)$



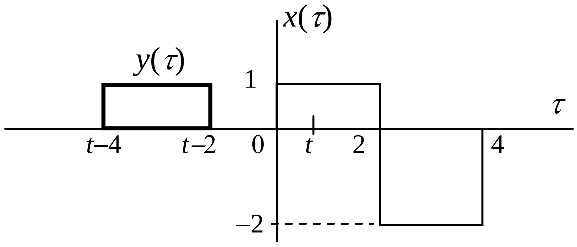
(D) Compute $\int_0^4 x(1-0.5t)\,y(2t+1)\delta(t-1)\,dt$. (2%)

Ans $\int_0^4 x(1-0.5t)\,y(2t+1)\delta(t-1)dt=\int_0^4 x(0.5)\,y(3)\delta(t-1)dt=\int_0^4 \delta(t-1)dt=1$

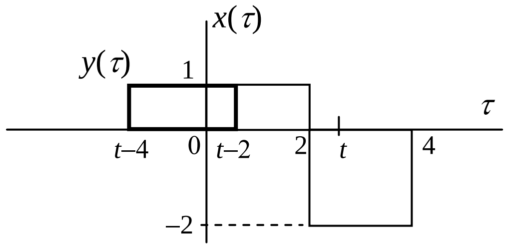
(E) Compute $x(t)*y(t)$ (10%)

Ans $x(t)*y(t)$

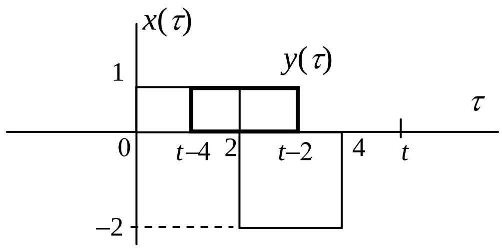
(1) $0<t<2$ $x(t)*y(t)=0$



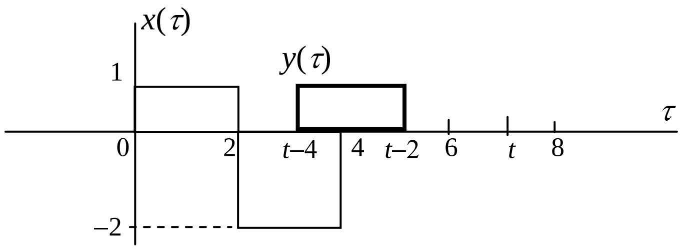
(2) $2<t<4$ $\qquad x(t)*y(t)=t-2$



(3) $4<t<6$ $\qquad x(t)*y(t)=14-3t$

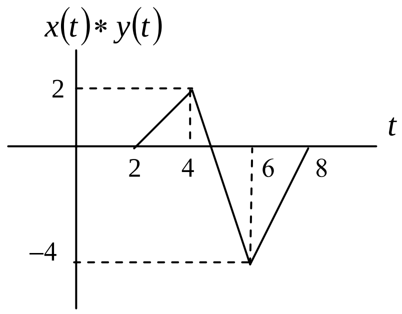


(4) $6<t<8$ $\qquad x(t)*y(t)=2t-16$

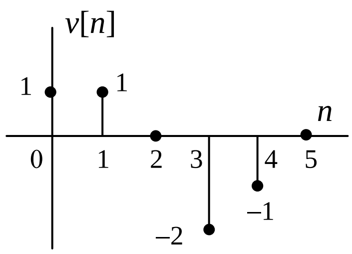


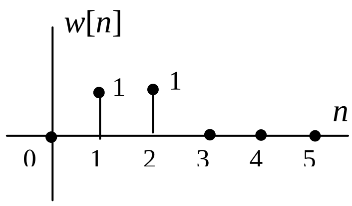
(5) $t>8$ $\qquad x(t)*y(t)=0$

Hence, from (1) to (5) we have



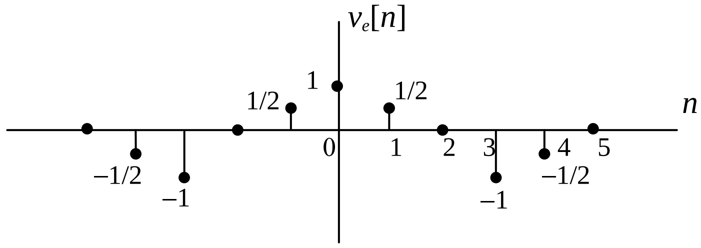
2. Consider v[n] and w[n] below:

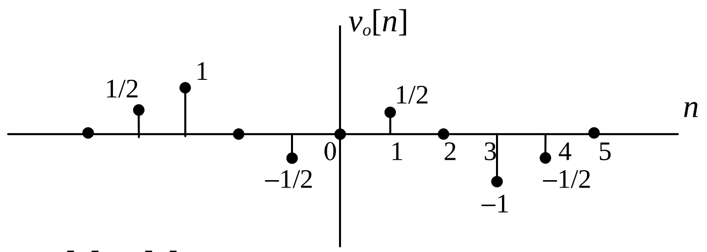




(A) Plot $v_e[n]$ and $v_o[n]$, which are the even and odd parts of $v[n]$. (4%)

Ans $\quad v_e=\frac{1}{2}\left(v[n]+v[-n]\right),\quad v_o=\frac{1}{2}\left(v[n]-v[-n]\right)$





(B) Compute $v[n]*w[n]$ **(8%)**

Ans

$$Let\ h[n]=v[n]*w[n]=\sum_{k=-\infty}^{\infty}v[k]w[n-k]$$

$$h[0]=\sum_{k=-\infty}^{\infty}v[k]w[-k]=0$$

$$h[1]=\sum_{k=-\infty}^{\infty}v[k]w[1-k]=1$$

$$h[2]=\sum_{k=-\infty}^{\infty}v[k]w[2-k]=2$$

$$h[3]=\sum_{k=-\infty}^{\infty}v[k]w[3-k]=1$$

$$h[4]=\sum_{k=-\infty}^{\infty}v[k]w[4-k]=-2$$

$$h[5]=\sum_{k=-\infty}^{\infty}v[k]w[5-k]=-3$$

$$h[6]=\sum_{k=-\infty}^{\infty}v[k]w[6-k]=-1$$

Because there is no overlapping between v[k] and w[n-k] when n<0 or n>6

So $h[n]=0$, when $n<0\ or\ n>6$

3. Consider the following periodic signal:

$$z(t)=2-sin(1.8t)+3sin(2.7t)+cos(1.8t)$$

(A) Find the fundamental period. (2%)

Ans

$$\because Let\ \frac{2\pi}{1.8}n_1=\frac{2\pi}{2.7}n_2\Rightarrow\frac{n_1}{n_2}=\frac{1.8}{2.7}=\frac{2}{3}\Rightarrow\frac{2\pi}{1.8}\cdot 2=\frac{2\pi}{2.7}\cdot 3=\frac{2\pi}{0.9}=fundamental\ period$$

(B) Express $z(t)$ using complex exponentials. (5%)

Ans

$$z(t) = 2e^{j0t} - \frac{e^{j1.8t} - e^{-j1.8t}}{2j} + 3\cdot\frac{e^{j2.7t} - e^{-j2.7t}}{2j} + 1\cdot\frac{e^{j1.8t} + e^{-j1.8t}}{2}$$

$$= 2e^{j0t} + (0.5+0.5j)e^{j1.8t} + (0.5-0.5j)e^{-j1.8t} + (-\frac{3}{2}j)e^{j2.7t} + (\frac{3}{2}j)e^{-j2.7t}$$

$$= (2e^{j0})e^{j0t} + (\frac{\sqrt{2}}{2}e^{j\frac{\pi}{4}})e^{j1.8t} + (\frac{\sqrt{2}}{2}e^{-j\frac{\pi}{4}})e^{-j1.8t} + (\frac{3}{2}e^{-j\frac{\pi}{2}})e^{j2.7t} + (\frac{3}{2}e^{j\frac{\pi}{2}})e^{-j2.7t}$$

(C) What are the Fourier series coefficients? (3%)

Ans

Fourier series coefficient: $2e^{j0}, \frac{\sqrt{2}}{2}e^{j\frac{\pi}{4}}, \frac{\sqrt{2}}{2}e^{-j\frac{\pi}{4}}, \frac{3}{2}e^{-j\frac{\pi}{2}}, \frac{3}{2}e^{j\frac{\pi}{2}}$

4. Consider $g(t)=sin(3t)$.

(A) Let the sampling period be T=0.2 sec. Is the sampled sequence $sin(3nT)$, n=0,1,2,…, periodic? Why? (3%)

Ans

$Let\ \sin(3nT) = \sin(3(N+n)T)\ ,\ N \in Z$

$\Rightarrow 3NT = 2k\pi \Rightarrow 3N \cdot 0.2 = 0.6N = 2k\pi, k \in Z$

$\because N = \frac{2k\pi}{0.6} \notin Z$

$\therefore it\ is\ not\ periodic$

(B) What are the frequencies of the sampled sequences $sin(3nT)$, n=0,1,2,…, for T= 0.4, and 0.8 sec? (4%)

Ans

$\omega_0 = 3$

$when$ T=0.4s

$\omega_{Ny} = \frac{\pi}{T} = \frac{\pi}{0.4} \cong 7.854$

$\because 3 \le 7.854 \Rightarrow the\ freqency\ of\ the\ sample\ sequence = 3\ \text{rad/s}$

$when$ T=0.8s

$\omega_{Ny} = \frac{\pi}{T} = \frac{\pi}{0.8} \cong 3.927$

$\because 3 \le 3.927 \Rightarrow the\ freqency\ of\ the\ sample\ sequence = 3\ \text{rad/s}$

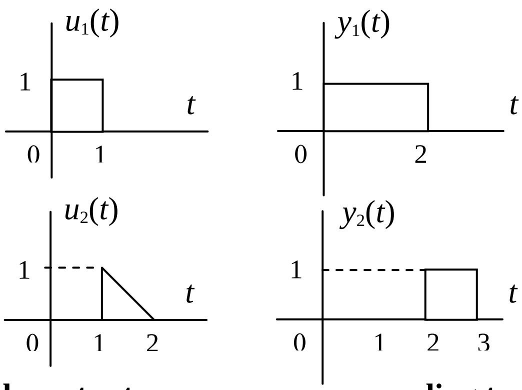
(C) Under what condition will the frequency of *sin*(3*nT*) equal 3 *rad*/*sec*? (4%)

Ans

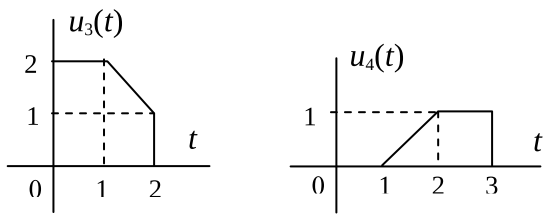
$$if\ \ \omega_0 \in (-\omega_{Ny}, \omega_{Ny}] = (-\frac{\pi}{T}, \frac{\pi}{T}],\ then\ the\ freqency\ of\ \ the\ sample\ sequence = \omega_0$$

$$\Rightarrow 3 \in (-\frac{\pi}{T}, \frac{\pi}{T}] \Rightarrow 3 \le \frac{\pi}{T} \Rightarrow \mathrm{T} \le \frac{\pi}{3} \cong 1.047$$

5. Let $y_1(t)$ and $y_2(t)$ be the output responses of $u_1(t)$ and $u_2(t)$, respectively. They are depicted as below:



Find the output responses corresponding to $u_3(t)$ and $u_4(t)$ given as



(8%, 8%)

Ans

(a)

$\because u_3(t) = 2u_1(t) + u_1(t-1) + u_2(t)$

$\therefore y_3(t) = 2y_1(t) + y_1(t-1) + y_2(t)$

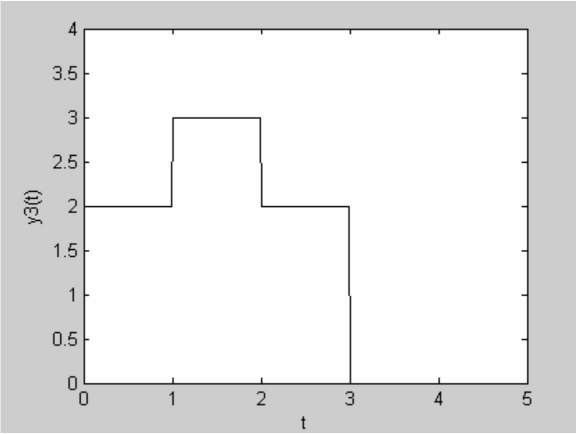
($graphic\ in$ Fig 5.1)

(b)

$\because u_4(t) = u_1(t-1) + u_1(t-2) - u_2(t)$

$\therefore y_4(t) = y_1(t-1) + y_1(t-2) - y_2(t)$

($graphic\ in$ Fig 5.2)



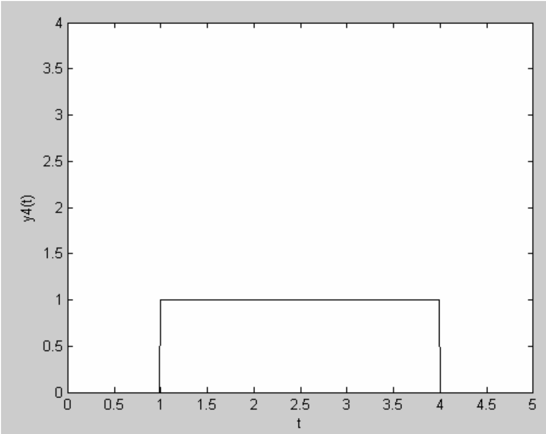


Fig 5.1

Fig 5.2

6. Compute the impulse responses of the following two systems:

(A) $y[n] = u[n] + 2u[n-2] - u[n-3] + u[n-5]$ **(3%)**

Ans

$n = 0: h[0] = \delta[0] + 2\delta[-2] - \delta[-3] + \delta[-5] = 1$

$n = 1: h[1] = \delta[1] + 2\delta[-1] - \delta[-2] + \delta[-4] = 0$

$n = 2: h[2] = \delta[2] + 2\delta[0] - \delta[-1] + \delta[-3] = 2$

$n = 3: h[3] = \delta[3] + 2\delta[1] - \delta[0] + \delta[-2] = -1$

$n = 4: h[4] = \delta[4] + 2\delta[2] - \delta[1] + \delta[1] = 0$

$n = 5: h[5] = \delta[5] + 2\delta[3] - \delta[2] + \delta[0] = 1$

$for\ n < 0\ or\ n > 5,\ h[n] = 0$

$it\ is$ an FIR filter

(B) $y[n] - 2y[n-1] = u[n-1] - u[n-2]$ **(5%)**

Ans

$n = 0: h[0] = 2h[-1] + \delta[-1] - \delta[-2] = 0$

$n = 1: h[1] = 2h[0] + \delta[0] - \delta[-1] = 1$

$n = 2: h[2] = 2h[1] + \delta[1] - \delta[0] = 2 - 1 = 1$

$n = 3: h[3] = 2h[2] = 2$

$n = 4: h[4] = 2h[3] = 4$

$n = 5: h[5] = 2h[4] = 8$

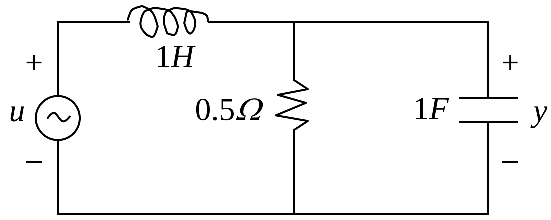
$for\ n < 0,\ h[n] = 0$

$for\ n > 5,\ h[n] = 2^{n-2}$

$it\ is$ an IIR filter

7. (A) Find a differential equation to describe the following circuit. (8%)

(B) Determine the steady-state response *y*(*t*) as *t*→∞ for *u*(*t*)=*sin*(*t*). (4%)



Ans

(A)

$$\begin{cases} u(t) = V_c(t) + L\dfrac{di_L(t)}{dt} \\ i_L(t) = \dfrac{V_c(t)}{0.5} + C\dfrac{dV_c(t)}{dt} \end{cases}$$

$$\Rightarrow u(t) = V_c(t) + 1\cdot\frac{d}{dt}\left(\frac{V_c(t)}{0.5} + 1\frac{dV_c(t)}{dt}\right)$$

$$= V_c(t) + 2\frac{dV_c(t)}{dt} + \frac{d^2V_c(t)}{dt^2}$$

$$\because V_c(t) = y(t)$$

$$\therefore u(t) = y(t) + 2y'(t) + y''(t)$$

(B)

$$\because it\ is\ in\ steady-state$$

$$\therefore y(t) = y_h(t) + y_p(t) = y_p(t) \quad (\, y_h(t) = 0\ )$$

$$u(t) = \sin(t)$$

$$\text{Let } y_p(t) = A\cos(t) + B\sin(t)$$

$$u(t) = y(t) + 2y'(t) + y''(t)$$

$$\Rightarrow u(t) = y_p(t) + 2y_p'(t) + y_p'(t)$$

$$\Rightarrow \sin(t) = (A\cos(t) + B\sin(t)) + 2(-A\sin(t) + B\cos(t)) + (-A\cos(t) - B\sin(t))$$

$$\Rightarrow \sin(\mathrm{t}) = (A + 2B - A)\cos(t) + (B - 2A - B)\sin(t)$$

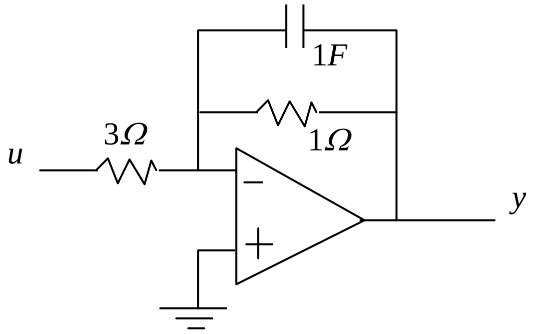
$$\Rightarrow 2B = 0, -2A = 1$$

$$\Rightarrow B = 0, A = -\frac{1}{2}$$

$$\Rightarrow y(t) = y_p(t) = -\frac{1}{2}\cos(t)$$

8. (A) Find a differential equation to describe the following circuit. (6%)

(B) Determine its step response with zero initial condition. (4%)



Ans

(A)

$$\begin{cases} i_{in}(t) = \dfrac{u(t)}{3} \\ i_{in}(t) + C\dfrac{dy(t)}{dt} + \dfrac{y(t)}{1} = 0 \end{cases}$$

$$\Rightarrow \frac{u(t)}{3} + \frac{dy(t)}{dt} + y(t) = 0$$

$$\Rightarrow u(t) = -3y'(t) - y(t)$$

(B)

$$Let\ u(t) = q(t),\ y(t) = y_h(t) + y_p(t)$$

$$for\ t \geq 0,\ \frac{dy(t)}{dt} + y(t) = -\frac{1}{3}$$

$$find\ y_h(t):\ let\ y_h(t) = Ae^{-ct}$$

$$char\ equ: c + 1 = 0 \Rightarrow c = -1$$

$$\Rightarrow y_h(t) = Ae^{-t}$$

$$find\ y_p(t):\ y_p(t) = -\frac{1}{3}$$

$$\Rightarrow y(t) = y_h(t) + y_p(t) = Ae^{-t} - \frac{1}{3}$$

$$\Rightarrow for\ zero\ initial\ condition:\ y(0^+) = 0$$

$$\Rightarrow\ y(0^+) = A - \frac{1}{3} = 0 \Rightarrow A = \frac{1}{3}$$

$$\Rightarrow y(t) = y_h(t) + y_p(t) = \frac{1}{3}e^{-t} - \frac{1}{3}$$

$$for\ steady-state: y(t) = -\frac{1}{3}$$