

Signals And Systems Final Exam

Note that no short answer is allowed! In your answer, you have to clearly show your reasons.

1. Consider the following systems:

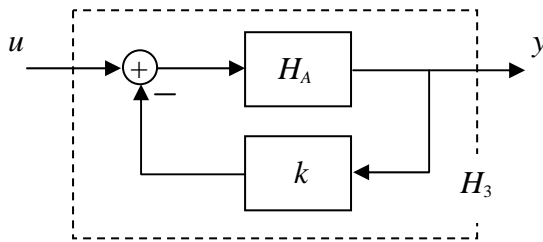
$$H_A = \frac{s^2 + 6s + 5}{s^3 + s^2 + 4s + 9}$$

$$H_B = \frac{s^3 + s^2 + 4s + 9}{s^4 + 9s^3 + 26s^2 + 34s + 20}$$

$$H_C = \frac{s^3 + 4s^2 + 4s + 8}{s^4 + 9s^3 + 26s^2 + 34s + 20}$$

(5%) (A) A stable system is called minimum phase if all its zeros are also located in the left-half complex plane. Which system is of minimum phase?

(10%) (B) A system H_3 with constant feedback k is depicted as below:



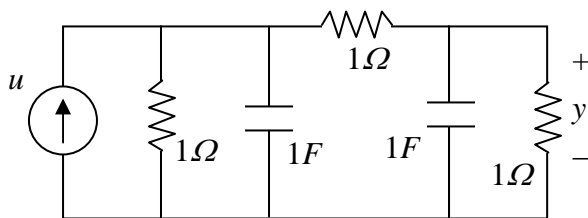
Its transfer function is found as

$$H_3 = \frac{H_A}{1 + kH_A}$$

If H_3 is stable, please determine the range of k .

(10%) (C) If $k=1$ for H_3 , please find its state-space description in one of the canonical form introduced in our class.

2. Consider the following circuit with input current u and output voltage y :



(8%) (A) Find its state-space description by choosing the two capacitor voltages as its state variables.

(15%) (B) Realize the state-space description by the use of operational amplifiers.

(5%) (C) Determine its transfer function.

3. For a stable LTI system with transfer function $H(s)$, let its input be a sinusoidal function given as $u = A \cos(\omega t)$.

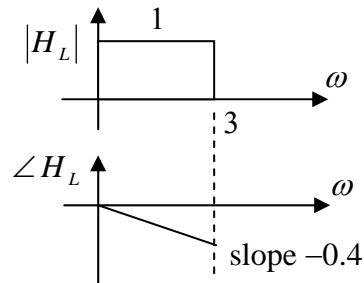
(8%) (A) What is the steady-state output? You have to show the way to obtain it.

(10%) (B) Let the input of H_3 in Problem 1(C) be given as

$$u = 2 \cos(2t) - \sin(t-1)$$

Please determine its steady state output.

4. Let $H_L(s)$ be an ideal low-pass filter shown as below:



(4%) (A) What is the transfer function of $H_L(s)$?

(6%) (B) Determine the steady-state output $y_{ss}(t)$ of $H_L(s)$ if the input is given as $u(t) = \sin(1.2t) + \cos(2.4t) + \cos(3.6t)$

(15%) (C) To get $y[n]$, the discrete signals of $y_{ss}(t)$, we apply the sampling technique. Please sketch the magnitude and phase spectra of $y[n]$ corresponding to each of the sampling times $T = \pi, \pi/2, \pi/3$ sec.

(4%) (D) With the use of $H_L(s)$ as a low-pass filter, is it possible to recover $y_{ss}(t)$ from the discrete signals $y[n]$ obtained in (C)?

Wish you all have
a happy summer vacation!