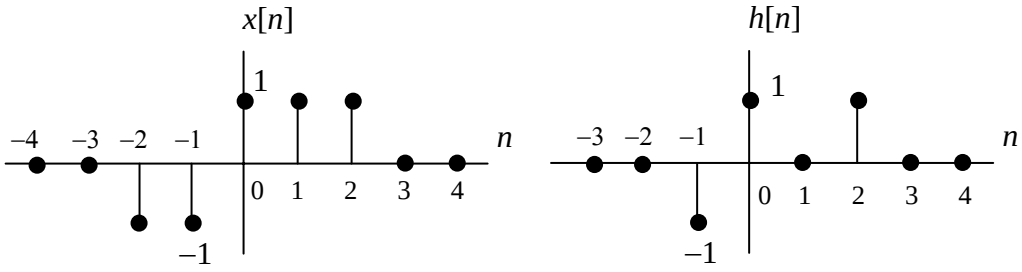
Signal and System, Exam#1 (Total score 105)

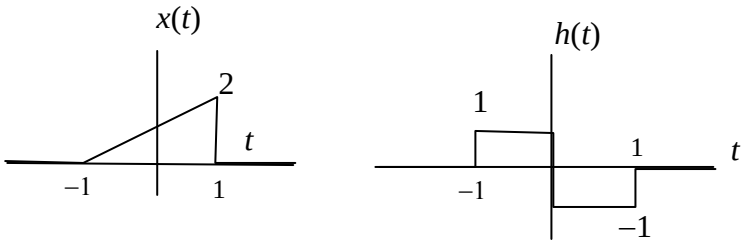
1. Let $x(t)=u(t+1)-t\cdot u(t-1)+t\cdot u(t-2)$. Find the closed-form expression of the following signals: (15%)

   (1) $\dfrac{dx(t)}{dt}$ (2) $\int_{-\infty}^{t}\left(x(\tau)\cdot u(\tau)\right)d\tau$ (3) $Od\{x(t)\}$

2. Find and sketch $y[n]=x[n]*h[n]$, where $x[n]$ and $h[n]$ are shown as (15%)



3. Find and sketch $y(t)=x(t)*h(t)$, where $x(t)$ and $h(t)$ are shown as (15%)



4. Solve the following differential equation for $t \geq 0$: (10%)

$$\ddot{y}(t)+5\dot{y}(t)+6y(t)=15os(t)-5sin(t),$$

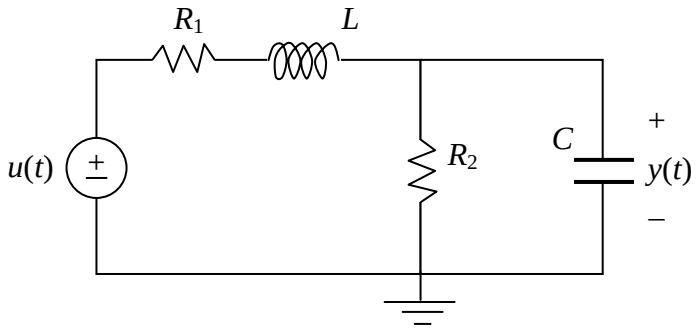
with $\dot{y}(0)=0$ and $y(0)=1$.

5. Consider the following LTI system:

$$2\dddot{y}(t)+\ddot{y}(t)+5\dot{y}(t)+6y(t)=\dot{x}(t)-5x(t)$$

(1) Draw the block diagram in direct-form-II structure. (15%)
(2) Determine its stability and show your reason. (5%)

6. Consider the following circuit with input voltage $u(t)$ and output voltage $y(t)$:



(1) Find its input-output equation, a second order differential equation. (10%)
(2) Determine its impulse response $h(t)$ if L=4H, C=0.25F, R_1=6Ω, R_2=1Ω. (10%)

7. Solve the following difference equation for $n \geq 0$: (10%)

$$y[n+2]+3y[n]+2y[n]=42\times 5^{n+1},$$

with $y[-2]=0$ and $y[-1]=1$.