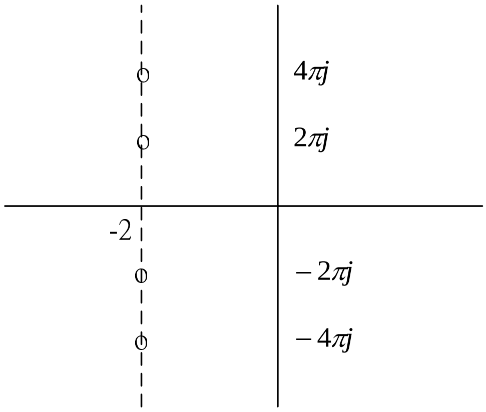
2005 Signals and Systems Final Solution

1. $X(s) = \dfrac{1 - e^{-(s+2)}}{s+2}$, $L[x(t-t_0)] = e^{-st_0} X(s)$ and $L[e^{-at}u(t)] = \dfrac{1}{s+a}$

   zeros: $1 - e^{-(s+2)} = 0$, $s = -2 \pm j2n\pi$, $n = 1, 2, \ldots$



2.(a) $X(s) = \dfrac{1}{s-1} + \dfrac{-(s+1)}{(s+1)^2+1} + \dfrac{-2}{(s+1)^2+1}$

$$x(t) = \begin{cases} -e^t u(-t) - e^{-t}(\cos t + 2\sin t)u(t), & -1 < \mathrm{Re}\{s\} < 1 \\ e^t u(t) - e^{-t}(\cos t + 2\sin t)u(t), & 1 < \mathrm{Re}\{s\} \end{cases}$$

(b) $-1 < \mathrm{Re}\{s\} < 1$: non-causal and stable

$1 < \mathrm{Re}\{s\}$: causal and unstable

3.(a) $u(t) = i''(t) + 2i'(t) + 2i(t)$

$$H(s) = \frac{1}{s^2 + 2s + 2}$$

(b) $\lim\limits_{t\to\infty} i(t) = \lim\limits_{s\to 0} sI(s)$, $U(s) = \dfrac{2}{s^2+4}$

$$\lim_{t\to\infty} i(t) = \lim_{s\to 0} s \times \frac{1}{s^2+2s+2} \times \frac{2}{s^2+4} = 0$$

(c) $U(s) = \dfrac{1}{s}$

$$\lim_{t\to\infty} i(t) = \lim_{s\to 0} s \times \frac{1}{s^2+2s+2} \times \frac{1}{s} = \frac{1}{2}$$

(d) $i'(t) = \dfrac{-R}{L} i(t) + \dfrac{1}{L} v(t)$

$$v'(t) = \frac{-1}{C} i(t) - \frac{1}{RC} v(t) + \frac{1}{RC} u(t)$$

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u$$

$$= \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

(e) $\lambda_1 = -1 + i$, $\lambda_2 = -1 - i$

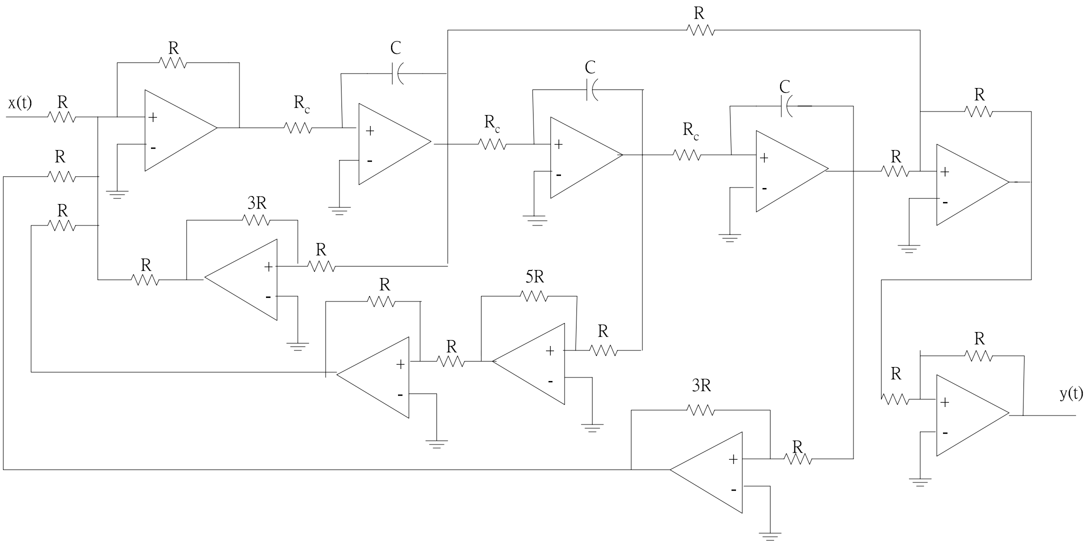
$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{v}_1 & \boldsymbol{v}_2 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} i \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} i \end{bmatrix}$$

$$e^{\boldsymbol{A}t} = \boldsymbol{P} \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix} \boldsymbol{P}^{-1}$$

4.(a) $\omega_b = 2$, $H(s) = \dfrac{8}{s^3 + 4s^2 + 8s + 8}$

(b) $y'''(t) + 4y''(t) + 8y'(t) + 8y(t) = 8u(t)$

5.



6. $H(s) = \dfrac{A(s)B(s)C(s)}{1 - A(s)B(s)E(s) - A(s)B(s)C(s)F(s) - B(s)C(s)D(s)}$