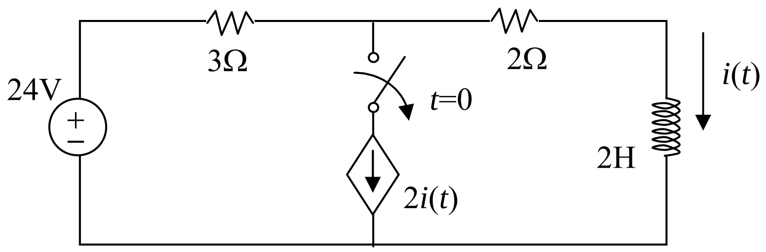
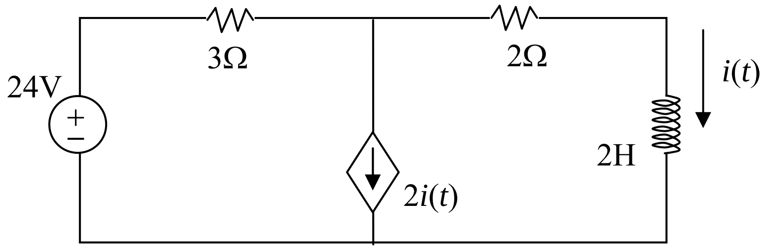
1. Suppose that the switch has been closed for a long time and is opened at t=0. Determine the inductor current $i(t)$ for t>0.



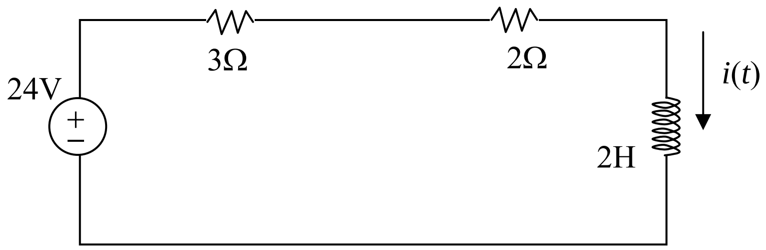
(12%)

Sol:



For t<0, we have

$$24 = 3\left(2i(0^-) + i(0^-)\right) + 2i(0^-) = 11i(0^-) \Rightarrow i(0^-) = \frac{24}{11}$$



For t>0, we have

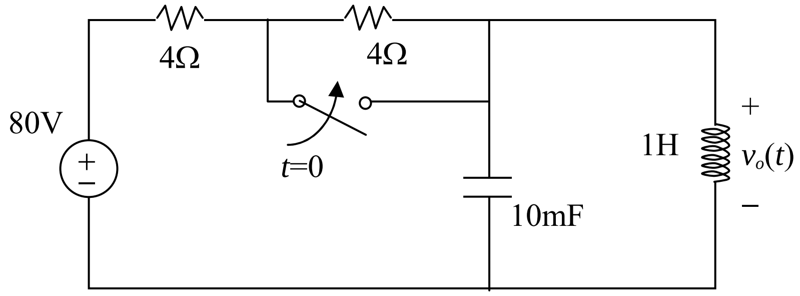
$$24 = (3+2)i(t) + 2\frac{di(t)}{dt} \Rightarrow \frac{di(t)}{dt} + \frac{5}{2}i(t) = 12$$

Hence, $i(t) = Ae^{-\frac{5}{2}t} + \frac{24}{5}$. Since $i(0) = i(0^-) = \frac{24}{11}$, we have

$$A + \frac{24}{5} = \frac{24}{11} \Rightarrow A = \frac{24}{11} - \frac{24}{5} = \frac{24\times(-6)}{55} = -\frac{144}{55}$$

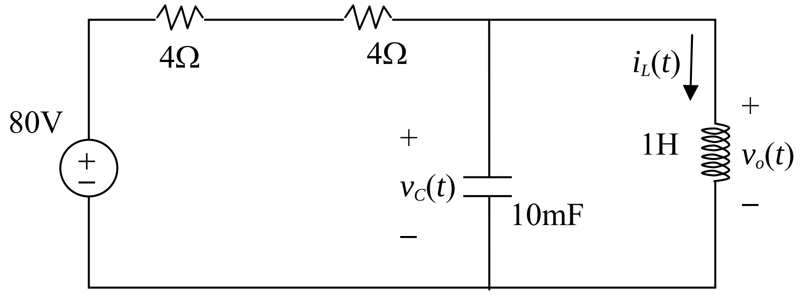
Therefore, $i(t) = -\frac{144}{55}e^{-\frac{5}{2}t} + \frac{24}{5}$ A .

2. Suppose that the switch has been opened for a long time and is closed at t=0. Find the output voltage $v_o(t)$ for t>0.

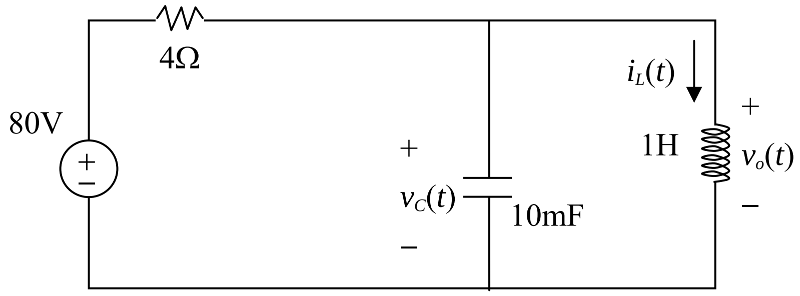


(15%)

Sol:



For t<0, we have $i_L(0^-) = \dfrac{80}{4+4} = 10$ and $v_C(0^-) = 0$ (1)



For t>0, we have $\dfrac{di_L(t)}{dt} = v_C(t)$ and $\dfrac{dv_C(t)}{dt} = 100\left(\dfrac{80 - v_C(t)}{4} - i_L(t)\right)$

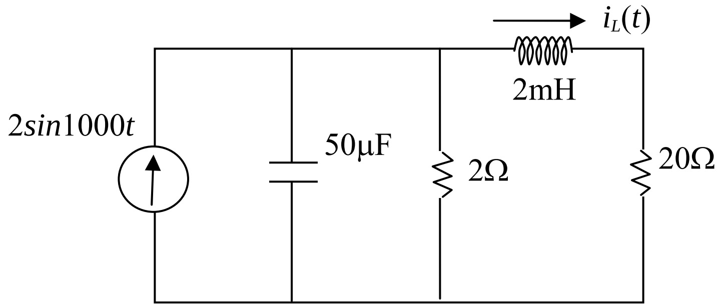
Therefore, from (1) we obtain $v_C(0^-) = 0$ and $\dot{v}_C(0^-) = 25 \times 80 - 100 \times 10 = 1000$. Besides,

$$\ddot{v}_C(t) = -25\dot{v}_C(t) - 100\frac{di_L(t)}{dt} = -25\dot{v}_C(t) - 100 v_C(t)$$

i.e., $\ddot{v}_C(t) + 25\dot{v}_C(t) + 100 v_C(t) = 0$. The eigenvalues satisfy $\lambda^2 + 25\lambda + 100 = 0$ or $\lambda = -5, -20$, which results in $v_C(t) = Ae^{-5t} + Be^{-20t}$ and $\dot{v}_C(t) = -5Ae^{-5t} - 20Be^{-20t}$. Thus, $A + B = 0$ and $-5A - 20B = 1000$. Then we have $A = \dfrac{200}{3}$ and $B = -\dfrac{200}{3}$.

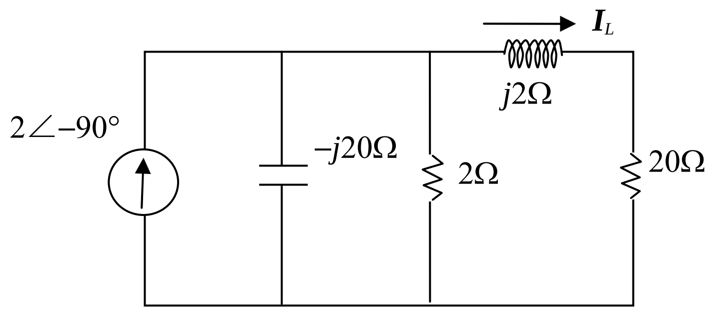
Hence, $v_o(t) = v_C(t) = \dfrac{200}{3}e^{-5t} - \dfrac{200}{3}e^{-20t}$.

3. Based on phasor method, determine the inductor current $i_L(t)$.



(15%)

Sol:



Based on phasor method, we have

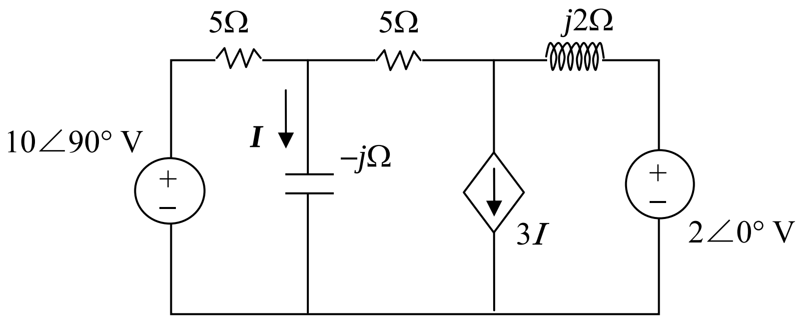
$$\boldsymbol{I}_L = 2\angle -90^\circ \times \left(\frac{\frac{1}{20+j2}}{\frac{j}{20}+\frac{1}{2}+\frac{1}{20+j2}} \right) = \frac{-j2}{(20+j2)\left(\frac{j}{20}+\frac{1}{2}\right)+1}$$

$$= \frac{-j40}{(20+j2)(j+10)+20} = -0.0326 - j0.1775 = 0.1805\angle -100.4^\circ$$

Hence, we obtain

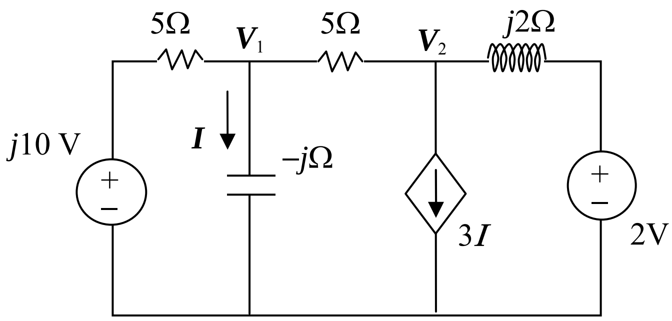
$$i_L(t) = 0.1805\cos(1000t - 100.4^\circ)$$

4. Solve the phasor current $\boldsymbol{I}$.



(15%)

Sol:



From nodal analysis, we have

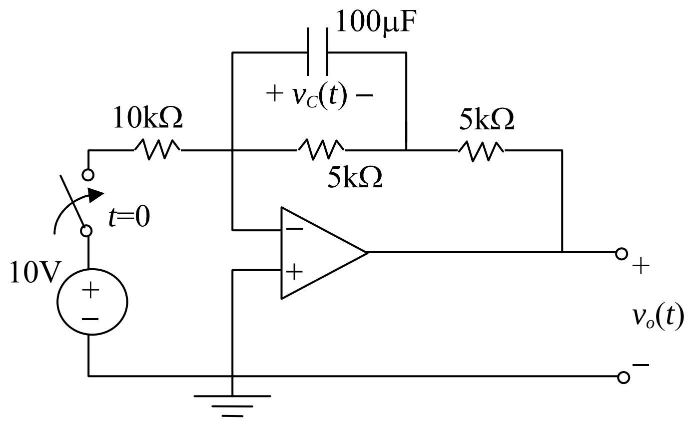
$$\frac{\boldsymbol{V}_1 - j10}{5} + \frac{\boldsymbol{V}_1}{-j} + \frac{\boldsymbol{V}_1 - \boldsymbol{V}_2}{5} = 0 \Rightarrow (2 + j5)\boldsymbol{V}_1 - \boldsymbol{V}_2 = j10$$

$$\frac{\boldsymbol{V}_2 - 2}{j2} + 3\frac{\boldsymbol{V}_1}{-j} + \frac{\boldsymbol{V}_2 - \boldsymbol{V}_1}{5} = 0 \Rightarrow (-2 + j30)\boldsymbol{V}_1 + (2 - j5)\boldsymbol{V}_2 = -j10$$

Hence, $\boldsymbol{V}_1$=1.0129 – j0.7551 and $\boldsymbol{V}_2$=5.8011– j 6.4457. Therefore, we have

$$\boldsymbol{I} = \frac{\boldsymbol{V}_1}{-j} = \frac{1.0129 - j0.7551}{-j} = 0.7551 + 1.0129j = 1.2634\angle 53.3^\circ$$

5. For the circuit with an ideal OP-amp, if the capacitor voltage is initially charged to be $v_C(0)$=1V, determine the output voltage $v_o(t)$ for t>0.



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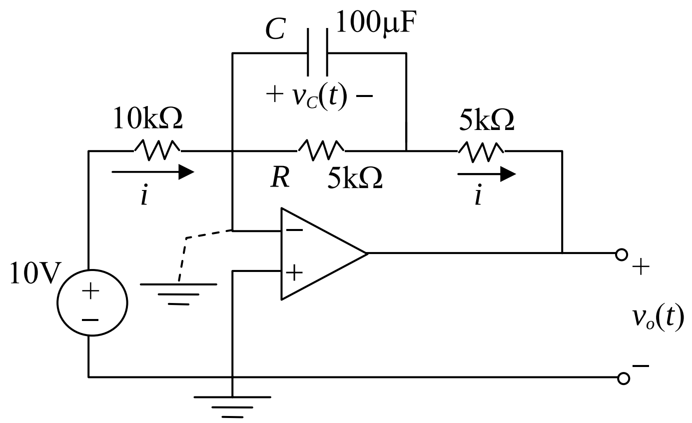
Sol:



For t>0, we have

$$i(t) = \frac{10}{10\text{k}} = 1\,\text{mA}$$

and from the KCL, we obtain

$$i(t) = C\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R}$$

$$\Rightarrow \frac{dv_C(t)}{dt} + \frac{v_C(t)}{RC} = \frac{i(t)}{C}$$

$$\Rightarrow \dot{v}_C(t) + 2v_C(t) = 10$$

$$\Rightarrow v_C(t) = 5 + Ae^{-2t}$$

Since $v_C(0) = 1$, we have A=–4.

Furthermore, $v_o(t) = -v_C(t) - 5000\times10^{-3} = -5 + 4e^{-2t} - 5 = 4e^{-2t} - 10$ V.

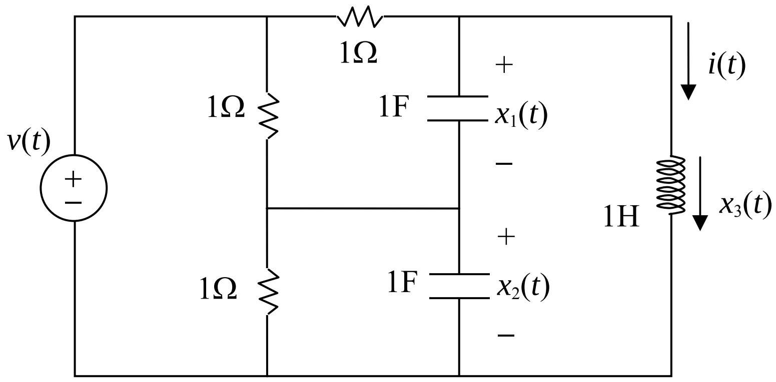
6. Consider the following third-order circuit with input voltage $v(t)$.

   (a) Write the state equation $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}v(t)$, where $\mathbf{x}(t) = [x_1(t)\ x_2(t)\ x_3(t)]^T$ contains three state variables x_1(t), x_2(t) and x_3(t) shown in the circuit. (10%)

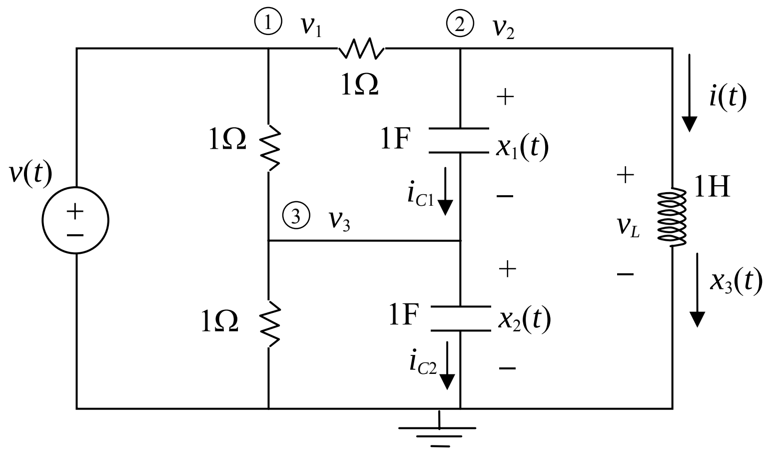
   (b) Let the inductor current be the output, i.e., $y(t)=i(t)=x_3$(t). Then, the circuit can be described by a third-order differential equation given as (6%)

   $$\dddot{y}(t) + a_2\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = b_2\ddot{v}(t) + b_1\dot{v}(t) + b_0 v(t)$$

   Find the coefficients a_i and b_i, i=0,1,2.



Sol:



(a)

From nodal analysis and by treating state variables as sources, we have node equations

$$v_1 = v(t)$$
$$v_2 = x_1(t) + x_2(t)$$
$$v_3 = x_2(t)$$

Based on the component equations, we have

$$\dot{x}_1(t) = i_{C1} = \frac{v_1 - v_2}{1} - i(t) = v(t) - (x_1(t) + x_2(t)) - x_3(t)$$
$$= -x_1(t) - x_2(t) - x_3(t) + v(t)$$
$$\dot{x}_2(t) = i_{C2} = i_1 + \frac{v_1 - v_3}{1} - \frac{v_3}{1}$$
$$= -x_1(t) - x_2(t) - x_3(t) + v(t) + v(t) - x_2(t) - x_2(t)$$
$$= -x_1(t) - 3x_2(t) - x_3(t) + 2v(t)$$
$$\dot{x}_3(t) = v_L = v_2 = x_1(t) + x_2(t)$$

Rearraging the above equations yields

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -3 & -1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} v(t)$$

(b)

Since $y(t)=i(t)=x_3(t)$, we have

$$y(t) = x_3(t)$$
$$\dot{y}(t) = \dot{x}_3(t) = x_1(t) + x_2(t)$$
$$\ddot{y}(t) = \dot{x}_1(t) + \dot{x}_2(t)$$
$$= -x_1(t) - x_2(t) - x_3(t) + v(t) - x_1(t) - 3x_2(t) - x_3(t) + 2v(t)$$
$$= -2x_1(t) - 4x_2(t) - 2x_3(t) + 3v(t)$$
$$\dddot{y}(t) = -2\dot{x}_1(t) - 4\dot{x}_2(t) - 2\dot{x}_3(t) + 3\dot{v}(t)$$
$$= 2x_1(t) + 2x_2(t) + 2x_3(t) - 2v(t)$$
$$+ 4x_1(t) + 12x_2(t) + 4x_3(t) - 8v(t) - 2x_1(t) - 2x_2(t) + 3\dot{v}(t)$$
$$= 4x_1(t) + 12x_2(t) + 6x_3(t) - 10v(t) + 3\dot{v}(t)$$

According to the characteristic equation

$$|\lambda \boldsymbol{I} - \boldsymbol{A}| = \begin{vmatrix} \lambda+1 & 1 & 1 \\ 1 & \lambda+3 & 1 \\ -1 & -1 & \lambda \end{vmatrix} = \lambda^3 + 4\lambda^2 + 4\lambda + 2$$

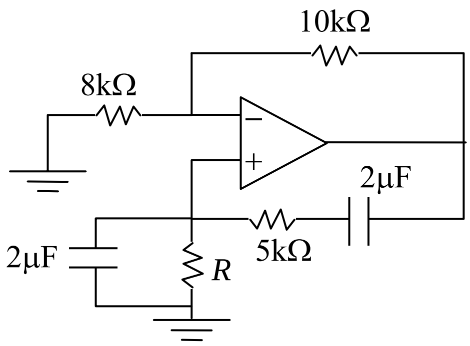
the third-order differential equation can be derived as

$$\dddot{y}(t) + 4\ddot{y}(t) + 4\dot{y}(t) + 2y(t) = 3\dot{v}(t) + 2v(t)$$

Hence, we obtain a_2=4, a_1=4, a_0=2, b_2=0, b_1=3, b_0=2.

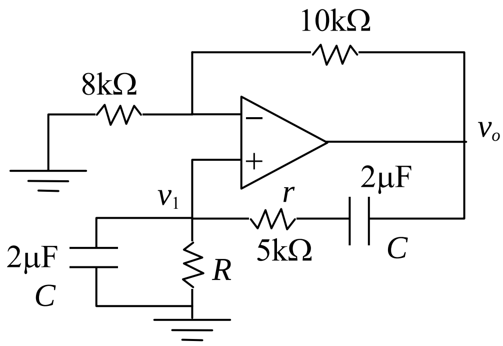
7. Complete the design of R in the following oscillator and calculate its oscillation frequency.

(15%)



Sol:

It is true that v_1 and v_o must be in phase. Therefore, their phasors must satisfy the following condition:



$$\frac{\mathbf{V}_1}{\mathbf{V}_o}=\frac{8}{10+8}=\frac{\dfrac{1}{R^{-1}+j\omega C}}{\dfrac{1}{R^{-1}+j\omega C}+r+\dfrac{1}{j\omega C}}$$

Hence,

$$\frac{4}{9}=\frac{\dfrac{R}{1+j\omega RC}}{\dfrac{R}{1+j\omega RC}+r+\dfrac{1}{j\omega C}}=\frac{R}{R+\left(r+\dfrac{1}{j\omega C}\right)(1+j\omega RC)}$$

$$\Rightarrow \frac{4}{9}=\frac{R}{2R+r+j\left(\omega rRC-\dfrac{1}{\omega C}\right)}$$

$$\Rightarrow \begin{cases} \dfrac{4}{9}=\dfrac{R}{2R+r} \\ \omega rRC-\dfrac{1}{\omega C}=0 \end{cases}$$

$$\Rightarrow R=4r=20\text{k}\Omega \quad \text{and} \quad \omega=\sqrt{\frac{1}{rRC^2}}=\frac{1}{2rC}=50$$

$$\Rightarrow R=20\text{k}\Omega \quad \text{and} \quad f=\frac{50}{2\pi}\text{ Hz}$$