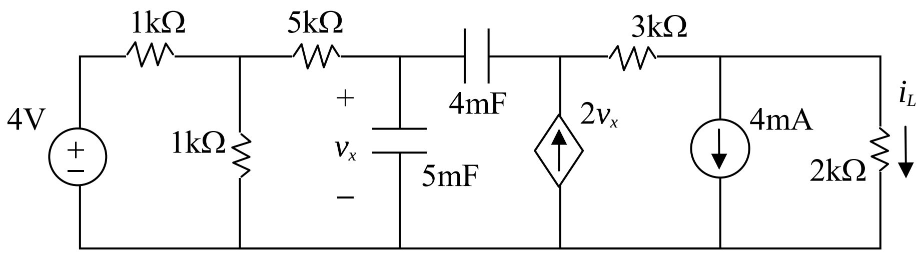
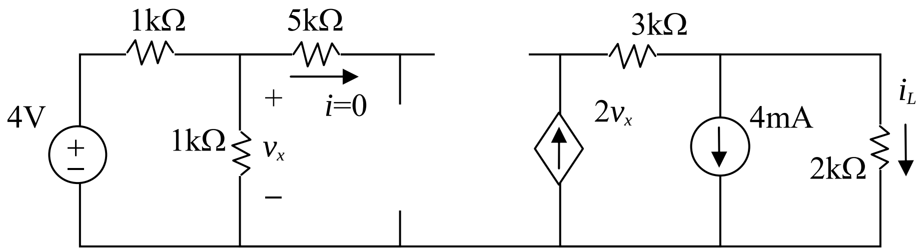
NCTU 2008 Course Electric Circuit(I)
Midterm-1 (Cap1 to Chap6)

1. Under dc condition, determine the load current i_L and the energy stored in the 5mF capacitor.

(12%)



Solution:



Under dc condition, we replace each capacitor with an open circuit. Then, no current through the 5kΩ resistor and the voltage across the 4kΩ resistor is v_x. According to the principle of voltage division, we obtain

$$v_x = 4 \times \frac{1}{1+1} = 2\text{ V}$$

Therefore, the load current is

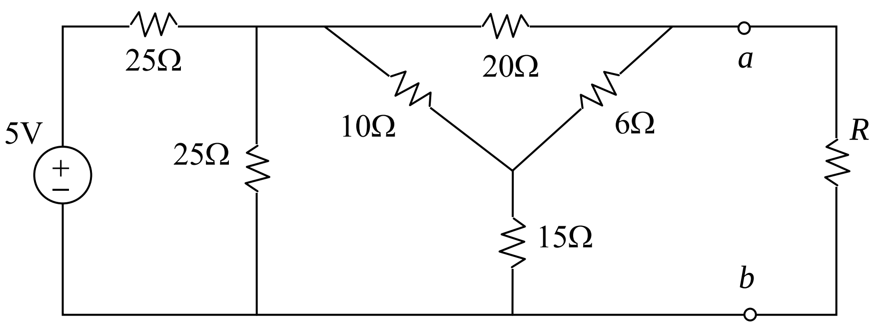
$$i_L = 2v_x - 4\times 10^{-3} = 2\times 2 - 4\times 10^{-3} = 3.996\text{A}$$

The energy stored in the 5mF capacitor is

$$E = \frac{1}{2}Cv_x^2 = \frac{1}{2}\times 5\times 10^{-3}\times 2^2 = 10\text{mJ}$$

2. Find the Thevenin equivalent circuit to the left of terminals *a* and *b*.

(15%)



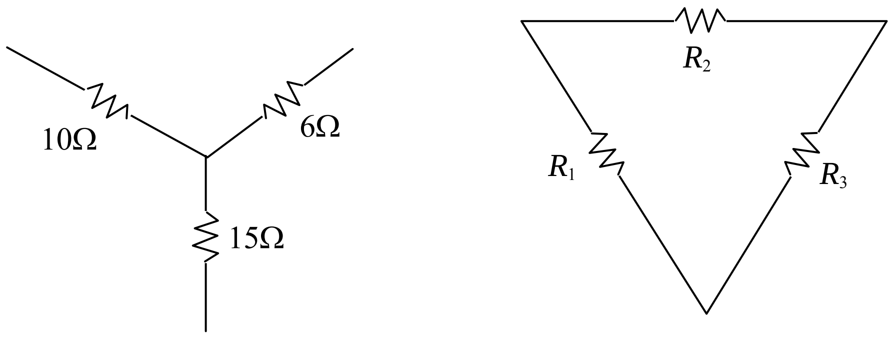
Solution:

According to the transformation formula, we have

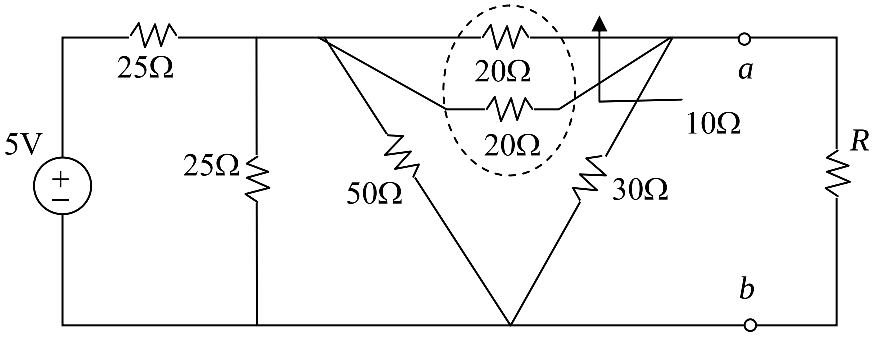
$$R_1 = \frac{10\times 6 + 6\times 15 + 15\times 10}{6} = \frac{300}{6} = 50\ \ \Omega$$

$$R_2 = \frac{10\times 6 + 6\times 15 + 15\times 10}{15} = \frac{300}{15} = 20 \quad \Omega$$

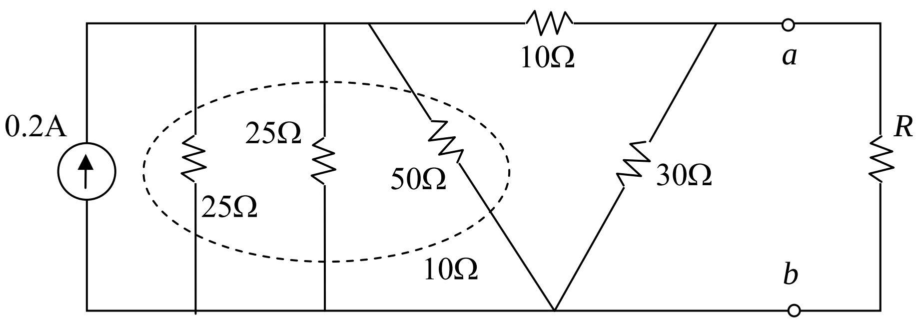
$$R_3 = \frac{10\times 6 + 6\times 15 + 15\times 10}{10} = \frac{300}{10} = 30 \quad \Omega$$



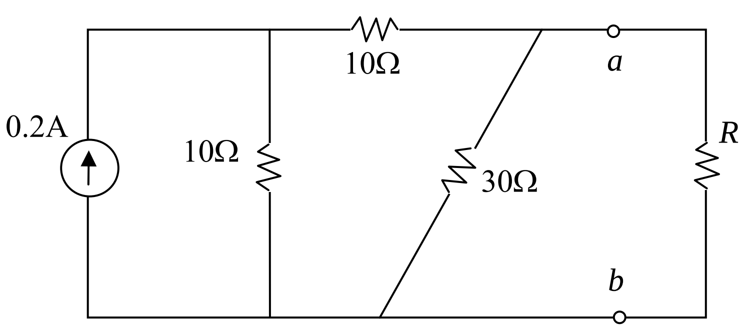
The circuit can be changed into



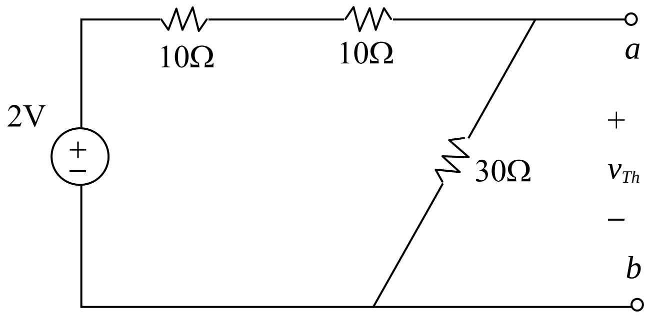
With source transformation, we have



i.e.



Further, we obtain



With $R\to\infty$, the Thevenin voltage is determined from the voltage across the terminals *a* and *b*, which is

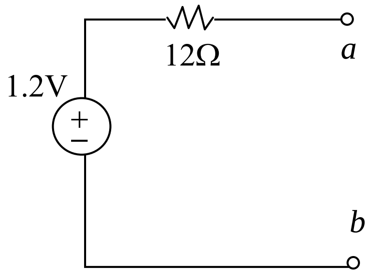
$$v_{Th} = \frac{30}{30+10+10}\times 2 = \frac{30}{50}\times 2 = 1.2 \quad \text{V}$$

By deleting the 2V source, the equivalent resistance to the left of terminals *a* and *b* is

$$R_{Th} = \frac{1}{\frac{1}{30}+\frac{1}{10+10}} = \frac{60}{2+3} = 12 \quad \Omega$$

Finally, the Thevenin circuit is shown

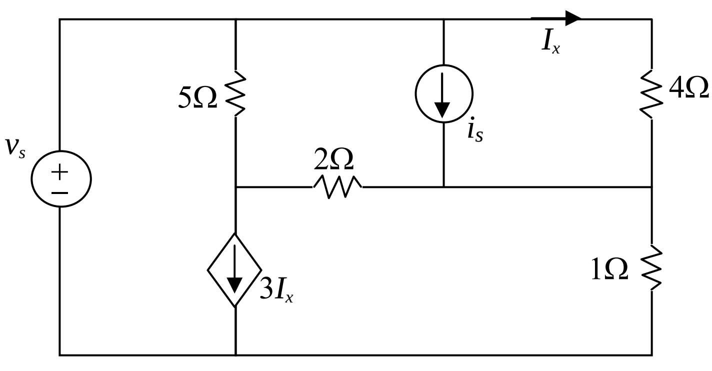
On the right.



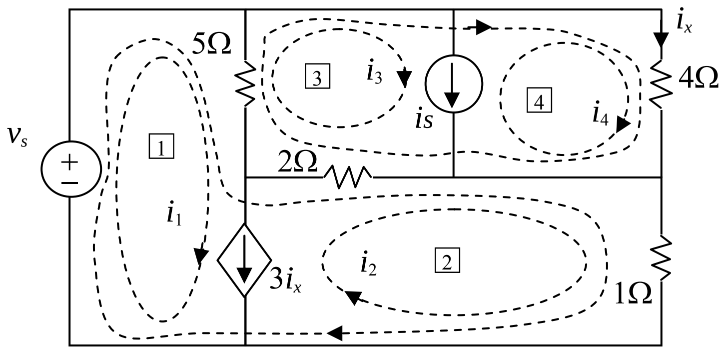
3. Based on the mesh-current method, write a set of mesh current equations in the following matrix form : $\boldsymbol{E}\cdot\boldsymbol{i} = \boldsymbol{B}\boldsymbol{u}$, where $\boldsymbol{E}$ is a 4×4 square matrix, $\boldsymbol{i} = [i_1 \quad i_2 \quad i_3 \quad i_4]^T$ is a 4×1 column vector containing all the mesh currents, $\boldsymbol{B}$ is a 4×2 matrix, and $\boldsymbol{u} = [v_s \quad i_s]^T$ is a 2×1 column vector composed of the independent sources.

   (*Note that you have to assign the mesh currents and number them from* i_1 *to* i_4 *in your own way.*)

(15%)



Solution:



First, number the mesh currents i_1, i_2, i_3, and i_4.

There are two supermesh, (1,2) and (3,4). The mesh current equations are

Supermesh 1 2:

KCL: $i_1 - i_2 = 3i_x = 3i_4 \quad \Rightarrow \quad i_1 - i_2 - 3i_4 = 0$ (1)

KVL: $-v_s + 5(i_1 - i_3) + 2(i_2 - i_3) + i_2 = 0$

$\Rightarrow \quad 5i_1 + 3i_2 - 7i_3 = v_s$ (2)

Supermesh 3 4:

KCL: $i_3 - i_4 = i_s$ (3)

KVL: $5(i_3 - i_1) + 4i_4 + 2(i_3 - i_2) = 0$

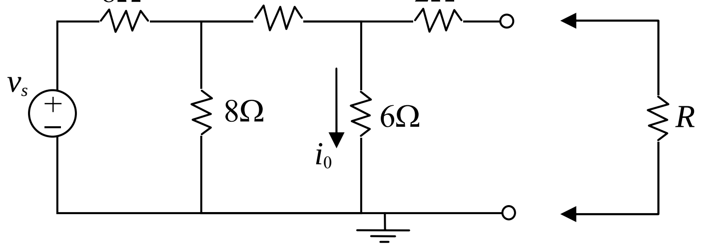
$\Rightarrow \quad -5i_1 - 2i_2 + 7i_3 + 4i_4 = 0$ (4)

From (1)-(4), we have

$$\underbrace{\begin{bmatrix} 1 & -1 & 0 & -3 \\ 5 & 3 & -7 & 0 \\ 0 & 0 & 1 & -1 \\ -5 & -2 & 7 & 4 \end{bmatrix}}_{\boldsymbol{E}} \cdot \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}}_{\boldsymbol{i}} = \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}}_{\boldsymbol{B}} \cdot \underbrace{\begin{bmatrix} v_s \\ i_s \end{bmatrix}}_{\boldsymbol{u}}$$

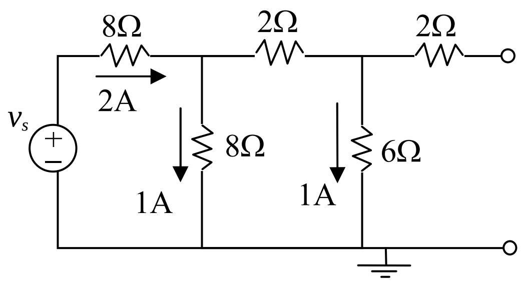
4. (A) When the load R is not connected, the current through 6Ω resistor is i_0=1A. Determine the source voltage v_s. (4%)

(B) If the load R is connected, what is the maximum power can be transferred to R? (8%)



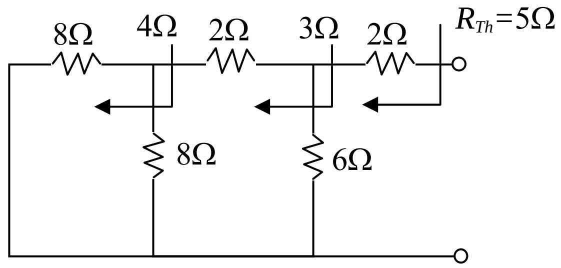
Solution:

(A) According to the principle of current division, the currents in the circuit are obtained as below:



Hence the voltage source is $v_s = 2\times 8 + 1\times 8 = 24$ V.

(B) By setting $R\to\infty$, the Thevenin voltage is the voltage across the 6Ω resistor, i.e., $v_{Th} = 1\times 6 = 6$ V. The equivalent resistance R_{Th} to the left of R is obtained by the following circuit:

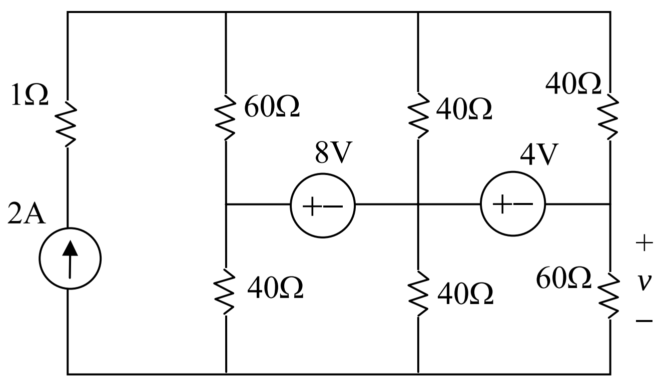


Since the equivalent resistance R_{Th}=5Ω, the maximum power transferred to the load is under the condition R=5Ω. Therefore, the maximum power is

$$P_{max} = v_R \cdot i_R = \left(\frac{1}{2}v_{Th}\right)\cdot\left(\frac{v_{Th}}{R+R_{Th}}\right) = \frac{6}{2}\cdot\frac{6}{5+5} = 1.8 \text{ J}.$$

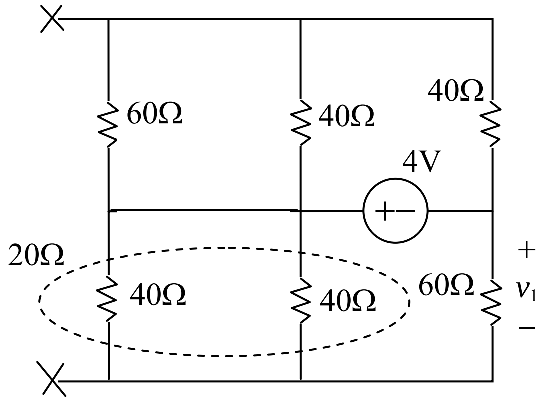
5. Use superposition to find v.

(12%)

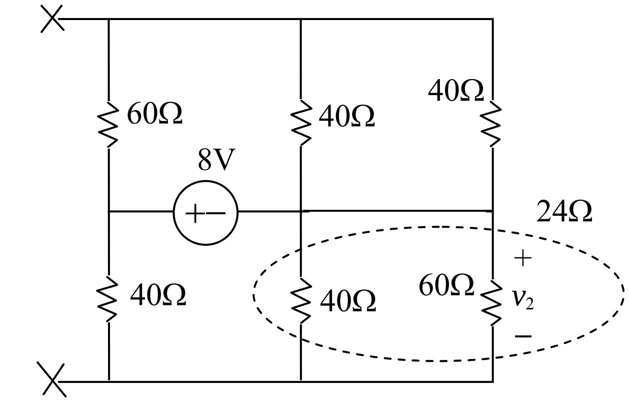


Solution:

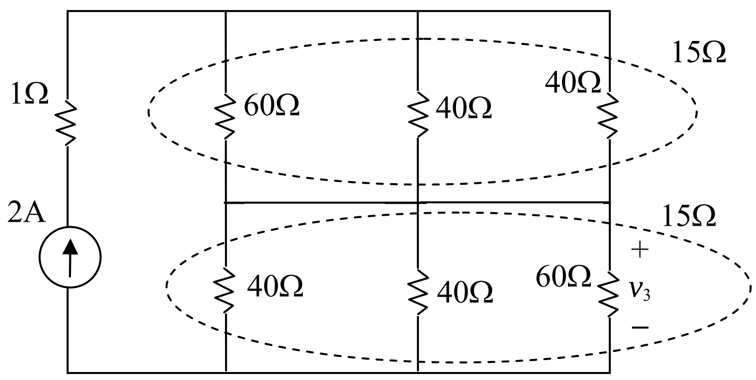
The voltage *v* can be solved by the following three cases:



The voltage $v_1 = -4\times\dfrac{60}{60+20} = -3$ V.



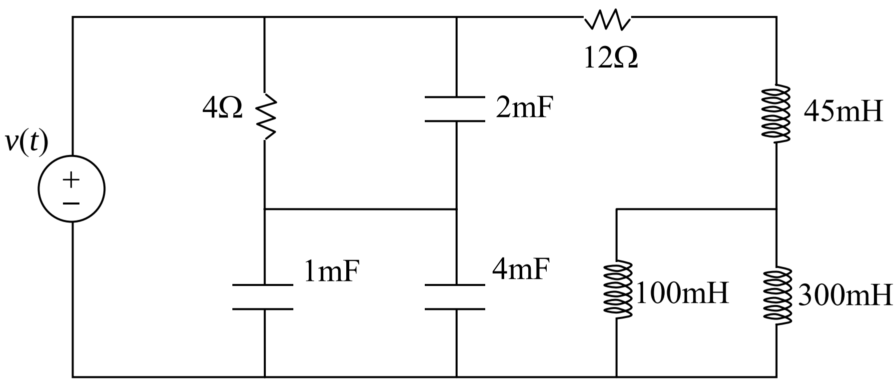
The voltage $v_2 = -8\times\dfrac{24}{40+24} = -3$ V.



The voltage $v_3 = 2\times 15 = 30$ V.

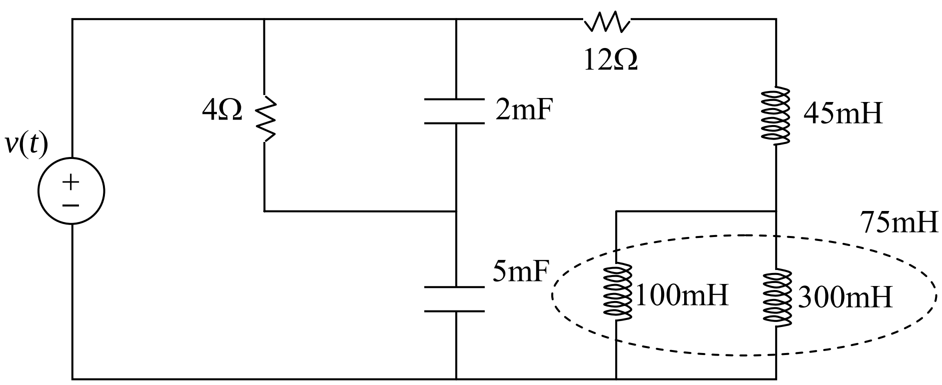
Hence, $v = v_1 + v_2 + v_3 = -3 - 3 + 30 = 24$ V.

6. Let $v(t) = 12 + 5e^{-2t}$ V. As $t\to\infty$, determine the total energy stored in the three capacitors and the total energy stored in the three inductors. (12%)

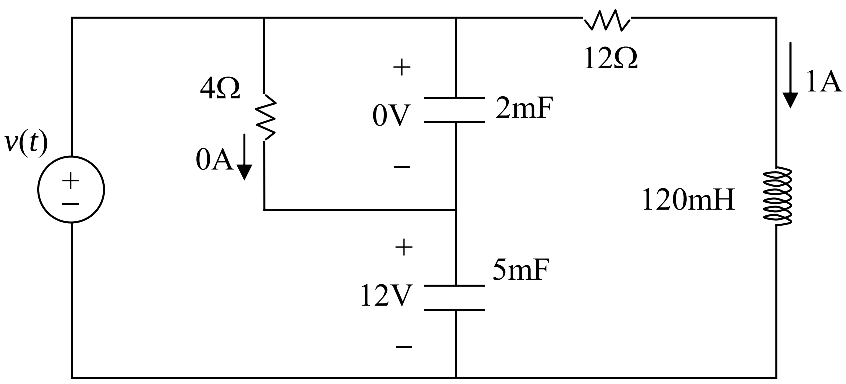


Solution:

First, rearrange the circuit as below:

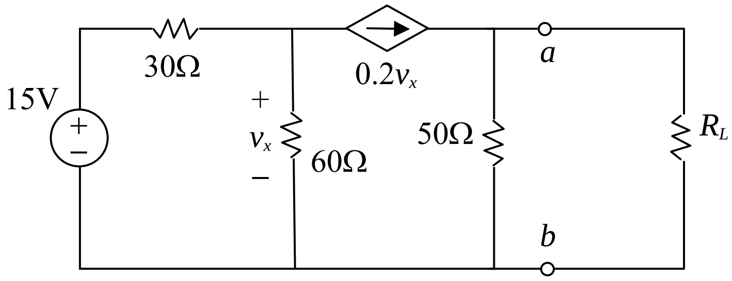


As $t \to \infty$, $v(t) = 12$, i.e., the circuit is under dc condition. Therefore, all the capacitors are open and all the inductors are short. The circuit is changed into



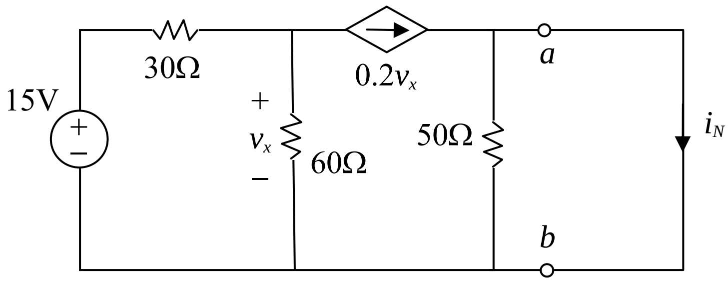
Hence, the total energy stored in the capacitors is $E_C = \frac{1}{2} \times 5 \times 10^{-3} \times 12^2 = 0.36$ J and the total energy stored in the inductors is $E_L = \frac{1}{2} \times 120 \times 10^{-3} \times 1^2 = 0.06$ J.

7. Find the Norton equivalent circuits with respect to terminals *a* and *b*. (12%)



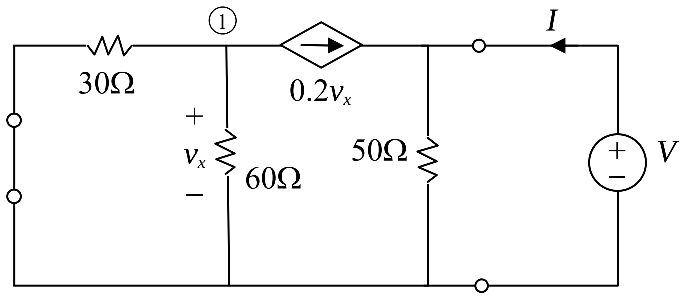
Solution:

First, find the Norton current i_N from the following circuit:

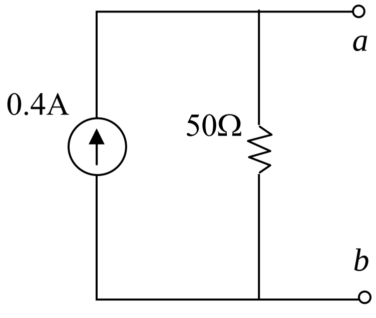


The voltage v_x can be calculated by $\frac{v_x - 15}{30} + \frac{v_x}{60} + 0.2v_x = 0 \Rightarrow v_x = 2$. The Norton current is $i_N = 0.2v_x = 0.4$ A.

The equivalent resistance is determined by adding an extra voltage source *V* as below:

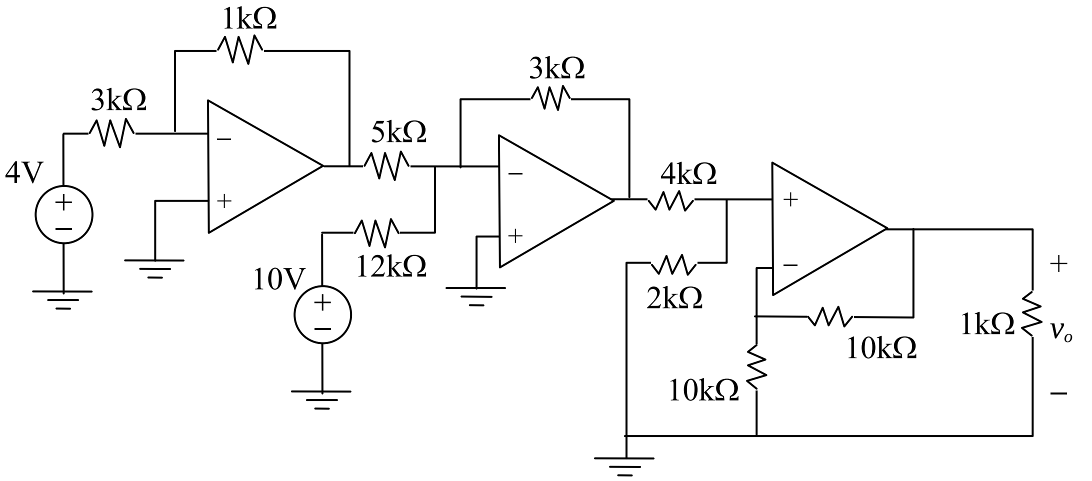


From node ①, we have $\frac{v_x}{30} + \frac{v_x}{60} + 0.2v_x = 0 \Rightarrow v_x = 0$. Hence, *V*=50*I*. That means the equivalent resistance $R_{eq} = \frac{V}{I} = 50\,\Omega$. The Norton circuit is shown as below:

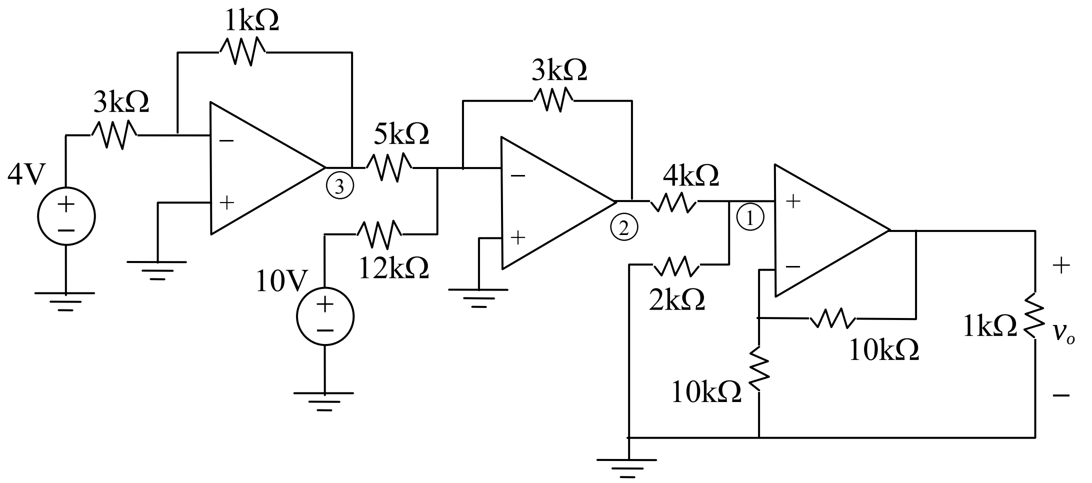


8. The following circuit contains ideal op amps, find the output voltage v_0.

(10%)



Solution:



Assign nodes ①, ② and ③ and their node voltages v_1, v_2, and v_3. Then, we have

$$v_3 = -\frac{1}{3}\times 4 = -\frac{4}{3}$$

$$v_2 = -\frac{3}{5}v_3 - \frac{3}{12}\times 10 = -\frac{3}{5}\times\left(-\frac{4}{3}\right) - \frac{5}{2} = -\frac{17}{10}$$

$$v_1 = \frac{2}{4+2}v_2 = \frac{1}{3}\times\left(-\frac{17}{10}\right) = -\frac{17}{30}$$

$$v_0 = 2v_1 = 2\times\left(-\frac{17}{30}\right) = -\frac{17}{15}\ \text{V}$$