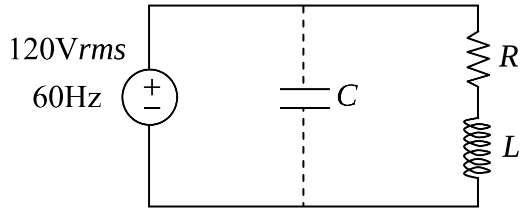
1. Consider the following circuit with the load R=10Ω and L=0.1H in series.



[6%] (a) What is the complex power delivered by the source?

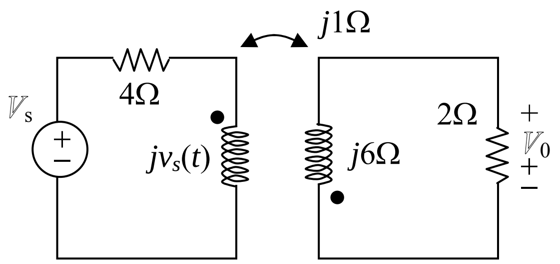
[3%] (b) What is the average power dissipated?

[3%] (c) What is the power factor of the load?

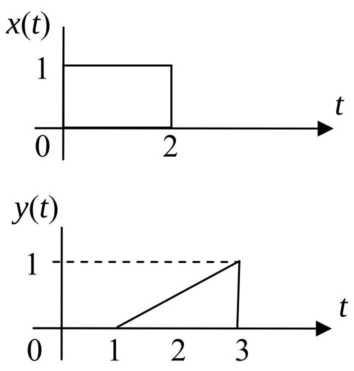
[6%] (d) Choose the capacitance C that will give a unity power factor?

2. In a balanced three phase Y-Y system, the source is an *abc* sequence of voltages and $\mathbb{V}_{an}$=120∠30° V*rms*. The line impedance per phase is 0.5+j1 Ω, while the per-phase impedance of the load is 10+j15 Ω. Calculate the three load voltages. [12%]

3. Let $\mathbb{V}_s$ =10∠60°. What is the phasor voltage $\mathbb{V}_0$? [12%]



4. Draw $z(t) = x(t) * y(t)$, the integral convolution of $x(t)$ and $y(t)$ [15%]

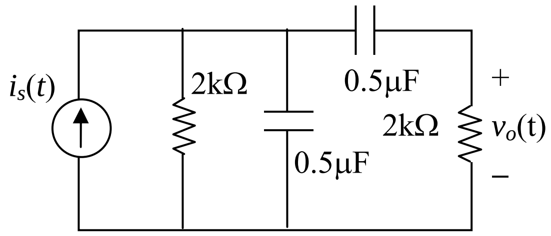


5. Neglect the initial conditions of the capacitors in the following circuit with input $i_s(t)$ and output $v_o(t)$.

[10%] (a) Determine the transfer function $H(s)$.

[6%] (b) Find its poles and zeros.

[8%] (c) If the output is $v_o(t)=V_o cos(\omega t-\pi/4)$ when $i_s(t)=cos\,\omega t$, what are the amplitude V_o and the frequency ω?



6. Determine the inverse Laplace transform of the following signals:

[4%] (a) $F(s) = \dfrac{3s+1}{(s+1)(s+4)}$

[5%] (b) $G(s) = \dfrac{s-1}{s(s+1)^2}$

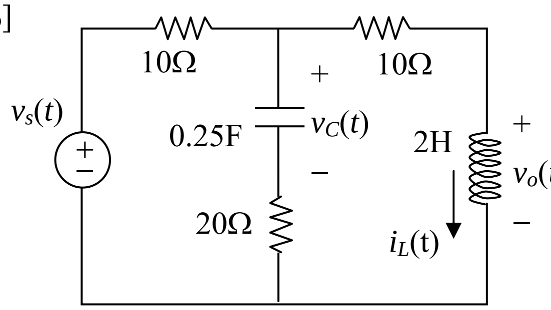
[5%] (c) $H(s) = \dfrac{2s^2-3}{(s+2)(s^2+2s+5)}$

7. Let $P(s) = \dfrac{s^2+3}{s^4-3s^2+2s}$ and determine the initial and final values of $p(t)$. [5%]

8. Consider the following circuit with a dc voltage source $v_s(t)$=10 V and initial conditions $v_C(0^-)$=1 V and $i_L(0^-)$=0 A .

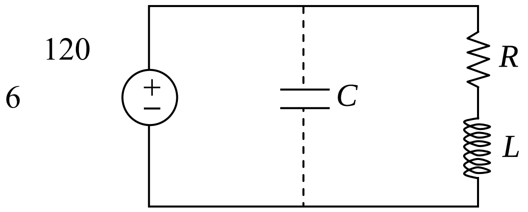
[15%] (a) Choose $v_C(t)$ and $i_L(t)$ as the state variables and write the state equation.

[15%] (b) Determine the output voltage $v_o(t)$.



Solution

1. Consider the following circuit with the load R=10Ω and L=0.1H in series.



   (a) What is the complex power delivered by the source?
   (b) What is the average power dissipated?
   (c) What is the power factor of the load?
   (d) Choose the capacitance C that will give a unity power factor?

Solution-01

The impedance of the load is

$$\boldsymbol{Z} = R + j\omega L = 10 + j60\times 0.1$$
$$= 10 + j6 = 11.66\angle 30.96^\circ$$

The current through the load is

$$\boldsymbol{I} = 120\angle 0^\circ / \boldsymbol{Z} = 120\angle 0^\circ / 11.66\angle 30.96^\circ$$
$$= 8.8235 - j5.2941 = 10.29\angle -30.96^\circ$$

(a) The complex power

$$\boldsymbol{S} = \boldsymbol{V}\boldsymbol{I}^* = 120\angle 0^\circ \times 10.29\angle 30.96^\circ$$
$$= 1234.8\angle 30.96^\circ = 1058.8 + j635.3\,(\text{VA})$$

(b) The average power is 1058.8 W.

(c) The power factor is

$$pf = cos\left(\frac{1058.8}{1234.8}\right) = 0.6544$$

(d) The complex power in C is

$$\boldsymbol{S} = \boldsymbol{V}\boldsymbol{I}^* = \boldsymbol{V}\left(j\omega C\boldsymbol{V}\right)^*$$
$$= -j\times 60\times 120^2\,C$$

To correct the power factor, we let

$60\times 120^2\,C = 635.3 \Rightarrow C = 735$

Hence, C=735μF.

2. In a balanced three phase Y-Y system, the source is an *abc* sequence of voltages and $\mathbb{V}_{an}$=120∠30° V*rms*. The line impedance per phase is 0.5+j1 Ω, while the per-phase impedance of the load is 10+j15 Ω. Calculate the three load voltages.

Solution-02

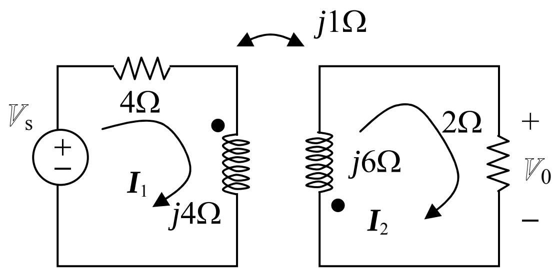
The load voltage $\boldsymbol{V}_{An}$ is

$$\boldsymbol{V}_{An} = \frac{10 + j15}{0.5 + j1 + 10 + j15}\boldsymbol{V}_{an}$$
$$= (0.9420 - j\,0.0068)\times 120\angle 30^\circ$$
$$= 0.9420\angle -0.415^\circ \times 120\angle 30^\circ$$
$$= 113.04\angle 29.585^\circ$$

Then the other two load voltages are

$$\boldsymbol{V}_{Bn} = 113.04\angle -90.415^\circ$$
$$\boldsymbol{V}_{Cn} = 113.04\angle 149.585^\circ$$

3. Let $\mathbb{V}_s$ =10∠60°. What is the phasor voltage $\mathbb{V}_0$?



Solution-03

The mesh equations are

$$\boldsymbol{V}_s = (4 + j4)\boldsymbol{I}_1 + j\boldsymbol{I}_2$$
$$0 = j\boldsymbol{I}_1 + (2 + j6)\boldsymbol{I}_2$$

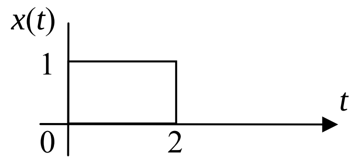
Hence, $\boldsymbol{I}_1 = j(2 + j6)\boldsymbol{I}_2 = (-6 + j2)\boldsymbol{I}_2$. Then

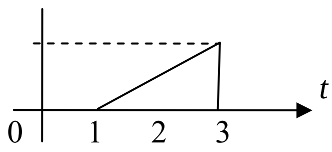
$$\boldsymbol{V}_s = (4 + j4)(-6 + j2)\boldsymbol{I}_2 + j\boldsymbol{I}_2$$
$$= (-32 - j15)\boldsymbol{I}_2$$

It results in $\boldsymbol{I}_2 = -\dfrac{\boldsymbol{V}_s}{32 + j15}$. Therefore,

$$\boldsymbol{V}_0 = 2\boldsymbol{I}_2 = -\frac{2\boldsymbol{V}_s}{32 + j15}$$
$$= -\frac{2\times 10\angle 60^\circ}{32 + j15} = 0.5659\angle -145.1^\circ$$

4. Draw $z(t) = x(t) * y(t)$, the integral convolution of $x(t)$ and $y(t)$





Solution-04

The integral convolution is defined as

$$z(t)=x(t)*y(t)=\int_{-\infty}^{\infty}x(\tau)y(t-\tau)d\tau$$

In this case, we have

$$z(t)=x(t)*y(t)=\int_{0}^{t}x(\tau)y(t-\tau)d\tau \quad \text{for } t>0.$$

For $0<t<1$, $z(t)=0$.

For $1<t<3$,

$$z(t)=\int_{0}^{t-1}1\cdot\left(\frac{t-1-\tau}{2}\right)d\tau$$

$$=\left.\frac{(t-1)\tau-0.5\tau^2}{2}\right|_{\tau=0}^{t-1}=\frac{(t-1)^2}{4}$$

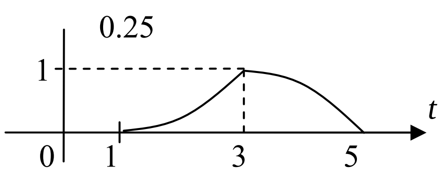
For $3<t<5$,

$$z(t)=\int_{t-3}^{2}1\cdot\left(\frac{t-1-\tau}{2}\right)d\tau$$

$$=\left.\frac{(t-1)\tau-0.5\tau^2}{2}\right|_{\tau=t-3}^{2}$$

$$=-0.25t^2+1.5t-1.25$$

For $t>5$, $z(t)=0$.

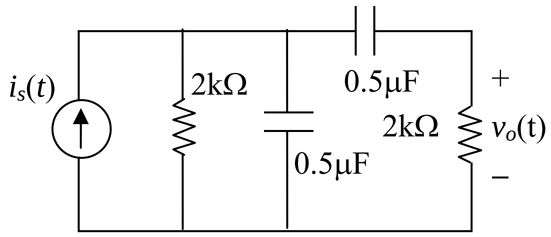


5. Neglect the initial conditions of the capacitors in the following circuit with input $i_s(t)$ and output $v_o(t)$.

   (a) Determine the transfer function $H(s)$.

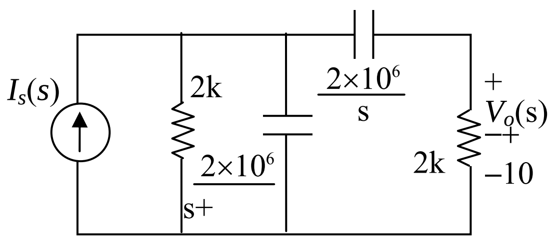
   (b) Find its poles and zeros.

   (c) If the output is $v_o(t)=V_o cos(\omega t-\pi/4)$ when $i_s(t)=cos\omega t$, what are the amplitude V_o and the frequency ω?



Solution-05

In s-form, we redraw the circuit as below:



(a) Let $G=1/(2k)$ and $C=0.5\mu$, then the output voltage is

$$V_o=\frac{1}{G}\cdot\frac{\dfrac{1}{\frac{1}{sC}+\frac{1}{G}}}{G+sC+\dfrac{1}{\frac{1}{sC}+\frac{1}{G}}}\cdot I_s$$

Hence,

$$H(s)=\frac{V_o}{I_s}=\frac{sC}{(G+sC)^2+sCG}$$

$$=\frac{2\times10^6 s}{s^2+3000s+10^6}$$

(b) Clearly, there is one zero at $s=0$. Solve $s^2+3000s+10^6=0$ and obtain the two poles at $s=-2618$ and -382.

(c) Since $H(j\omega)=\dfrac{j\omega 2\times10^6}{G(j\omega)}$, where

$$G(j\omega)=(j\omega)^2+3000(j\omega)+10^6 \\ =10^6-\omega^2+j3000\omega ,$$

the phase of $H(j\omega)$
is $90°-\angle G(j\omega)=-45°$.
Hence, $\angle G(j\omega)=135°$,
i.e., $3000\omega=\omega^2-10^6$.
Then, ω=3302.8 rad/s.
Thus,

$$V_o=|H(j3302.8)|=471.4024 .$$

6. Determine the inverse Laplace transform of the following signals:
   (a) $F(s)=\dfrac{3s+1}{(s+1)(s+4)}$
   (b) $G(s)=\dfrac{s-1}{s(s+1)^2}$
   (c) $H(s)=\dfrac{2s^2-3}{(s+2)(s^2+2s+5)}$

Solution-06

(a)

$$F(s)=\frac{3s+1}{(s+1)(s+4)}=-\frac{2/3}{s+1}+\frac{11/3}{s+4}$$

Hence, $f(t)=-\dfrac{2}{3}e^{-t}+\dfrac{11}{3}e^{-4t}$ for t>0.

(b)

$$G(s)=\frac{s-1}{s(s+1)^2}=\frac{-1}{s}+\frac{1}{s+1}+\frac{2}{(s+1)^2}$$

Hence, $g(t)=-1+e^{-t}+2te^{-t}$ for t>0.

(c)

$$H(s)=\frac{2s^2-3}{(s+2)(s^2+2s+5)} \\ =\frac{1}{s+2}+\frac{(s+1)+2(-5/2)}{(s+1)^2+2^2}$$

Hence, $h(t)=e^{-2t}+e^{-t}\left(\cos 2t-\dfrac{5}{2}\sin 2t\right)$

for t>0.

7. Let $P(s)=\dfrac{s^2+3}{s^4-3s^2+2s}$ and determine the initial and final values of $p(t)$.

Solution-07

For the initial value, we have

$$p(0)=\lim_{s\to\infty} sP(s)=\lim_{s\to\infty}\frac{s(s^2+3)}{s^4-3s^2+2s} \\ =\lim_{s\to\infty}\frac{s^2+3}{s^3-3s+2}=0$$

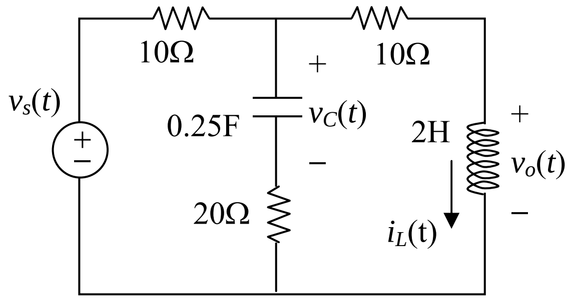
Since there exist poles located in the right half complex plane, it is unbounded for $t\to\infty$ and the final value theorem is not suitable to obtain $p(\infty)$. It must be solved as below:

$$P(s)=\frac{s^2+3}{s^4-3s^2+2s}=\frac{s^2+3}{s(s+2)(s-1)^2} \\ =\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s-1}+\frac{4/3}{(s-1)^2} ,$$

Hence, $p(t)=A+Be^{-2t}+Ce^{t}+\dfrac{4}{3}te^{t}$ for t>0.

Thus, $p(\infty)=\infty$.

8. Consider the following circuit with a dc voltage source $v_s(t)$=10 V and initial conditions $v_C(0^-)$=1 V and $i_L(0^-)$=0 A .
   (a) Choose $v_C(t)$ and $i_L(t)$ as the state variables and write the state equation.
   (b) Determine the output voltage $v_o(t)$.



Solution-08

(a) With mesh analysis, the mesh current equation for mesh 1 is

$$10i_1(t)+v_C(t)+20(i_1(t)-i_2(t))=v_s(t)$$

and the mesh current equation for mesh 2 is

$$i_2(t)=i_L(t)$$

Clearly, replacing $i_2(t)$ with $i_L(t)$ for mesh 1 yields

$$i_1(t) = -\frac{1}{30}v_C(t) + \frac{2}{3}i_L(t) + \frac{1}{30}v_s(t)$$

The component equation of the capacitor is

$$0.25\frac{dv_C(t)}{dt} = i_1(t) - i_2(t)$$

$\Rightarrow$

$$\frac{dv_C(t)}{dt} = 4(i_1(t) - i_2(t))$$

$$= 4\left(-\frac{1}{30}v_C(t) + \frac{2}{3}i_L(t) + \frac{1}{30}v_s(t) - i_L(t)\right)$$

$$= -\frac{2}{15}v_C(t) - \frac{4}{3}i_L(t) + \frac{2}{15}v_s(t)$$

The component equation of the inductor is

$$2\frac{di_L(t)}{dt} = v_s(t) - 10i_1(t) - 10i_2(t)$$

$\Rightarrow$

$$\frac{di_L(t)}{dt} = 0.5v_s(t) - 5i_1(t) - 5i_2(t)$$

$$= 0.5v_s(t) - 5(i_1(t) + i_2(t))$$

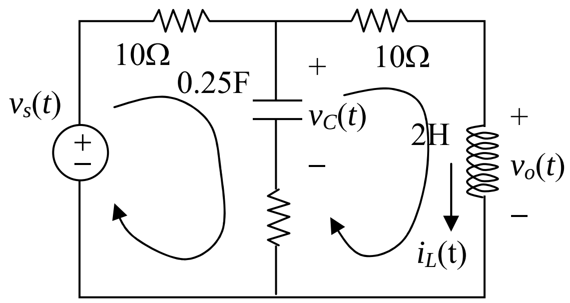
$$= 0.5v_s(t)$$

$$-5\left(-\frac{1}{30}v_C(t) + \frac{2}{3}i_L(t) + \frac{1}{30}v_s(t) + i_L(t)\right)$$

$$= \frac{1}{6}v_C(t) - \frac{25}{3}i_L(t) + \frac{1}{3}v_s(t)$$

Hence, the state equation in matrix form is

$$\begin{bmatrix} \frac{dv_C(t)}{dt} \\ \frac{di_L(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{2}{15} & -\frac{4}{3} \\ \frac{1}{6} & -\frac{25}{3} \end{bmatrix} \cdot \begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} \frac{2}{15} \\ \frac{1}{3} \end{bmatrix} v_s(t)$$



(b) Taking Laplace transform of the state equation, we have

$$s\begin{bmatrix} V_C(s) \\ I_L(s) \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2}{15} & -\frac{4}{3} \\ \frac{1}{6} & -\frac{25}{3} \end{bmatrix} \cdot \begin{bmatrix} V_C(s) \\ I_L(s) \end{bmatrix} + \begin{bmatrix} \frac{2}{15} \\ \frac{1}{3} \end{bmatrix} \frac{10}{s}$$

i.e.,

$$\begin{bmatrix} s+\frac{2}{15} & \frac{4}{3} \\ -\frac{1}{6} & s+\frac{25}{3} \end{bmatrix} \cdot \begin{bmatrix} V_C(s) \\ I_L(s) \end{bmatrix} = \begin{bmatrix} \frac{2}{15} \\ \frac{1}{3} \end{bmatrix} \frac{10}{s} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_C(s) \\ I_L(s) \end{bmatrix} = \begin{bmatrix} s+\frac{2}{15} & \frac{4}{3} \\ -\frac{1}{6} & s+\frac{25}{3} \end{bmatrix}^{-1} \begin{bmatrix} 1+\frac{20}{15s} \\ \frac{10}{3s} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_C(s) \\ I_L(s) \end{bmatrix} = \frac{\begin{bmatrix} s+\frac{25}{3} & -\frac{4}{3} \\ \frac{1}{6} & s+\frac{2}{15} \end{bmatrix} \begin{bmatrix} 1+\frac{20}{15s} \\ \frac{10}{3s} \end{bmatrix}}{s^2 + \frac{127}{15}s + \frac{4}{3}}$$

$$\Rightarrow I_L(s) = \frac{\frac{7}{2} + \frac{2}{3s}}{s^2 + \frac{127}{15}s + \frac{4}{3}}$$

$$= \frac{0.5}{s} + \frac{-0.4198}{s+8.3061} + \frac{-0.0803}{s+0.1605}$$

Taking the inverse Laplace transform, we obtain

$$i_L(t) = 0.5 - 0.4198e^{-8.3061t} - 0.0803e^{-0.1605t}$$

Hence, the voltage

$$v_o(t) = 2\frac{di_L(t)}{dt}$$

$$= 6.9738e^{-8.3061t} + 0.0258e^{-0.1605t}$$