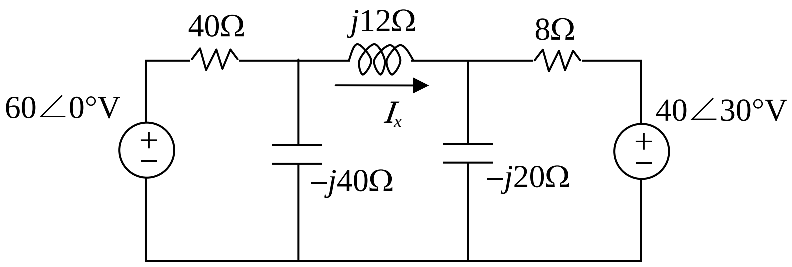
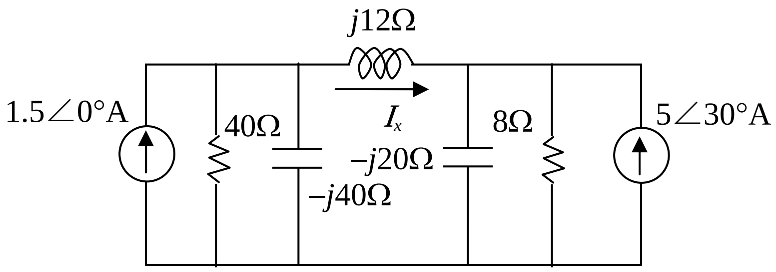
國立交通大學 96 暑修課程 電路學(一) Quiz-3 (Cap10 to Chap11 & Section 17.2)

1. Determine current I_x in the following circuit. (20%)

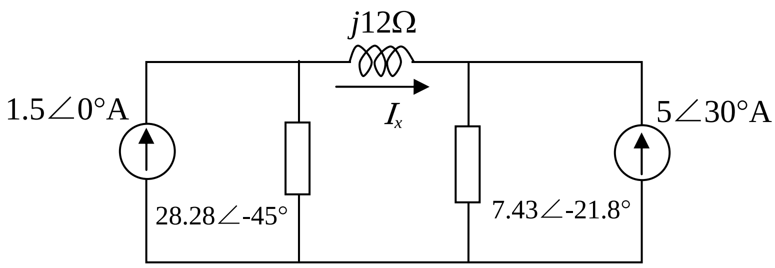


Solution:

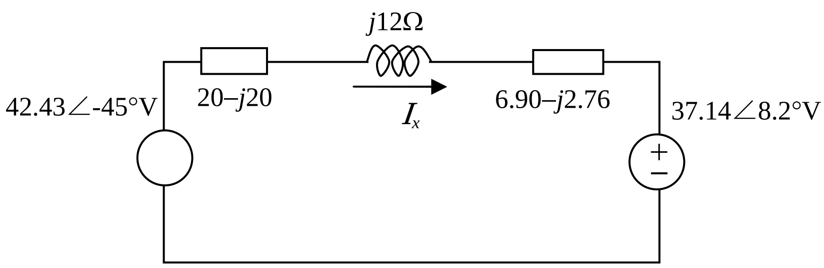
Use source transformation, the circuit is redrawn as below:



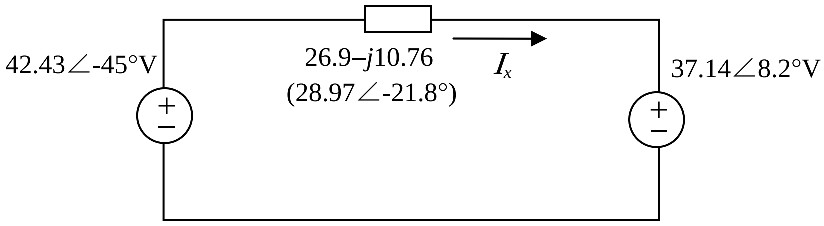
Further, it is changed as below:



Then, we have



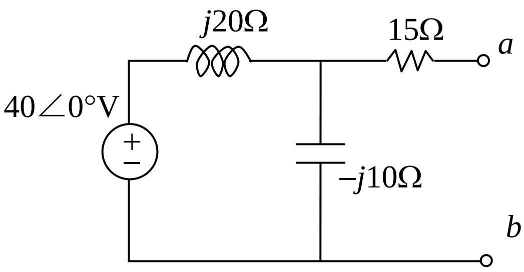
i.e.,



Hence,

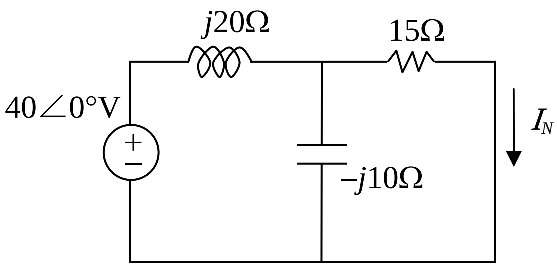
$$I_x = \frac{42.43\angle -45^\circ - 37.14\angle 8.2^\circ}{28.97\angle -21.8^\circ} = 0.24 - j1.22 \text{ A} = 1.24\angle -79^\circ \text{ A}$$

2. Find the Norton equivalent circuit at terminals *a*-*b*. (20%)



Solution:

The Norton current I_N is the current passing the short path from *a* to *b* on the right:

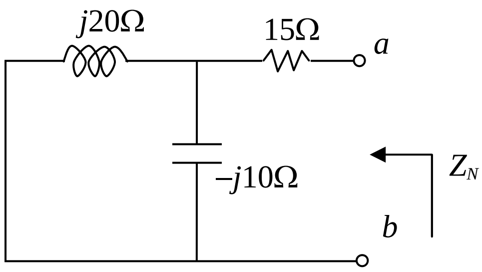


Hence,

$$I_N = \frac{-j10}{15-j10} \times \frac{40}{j20 + \dfrac{1}{\dfrac{1}{15} + \dfrac{1}{-j10}}}$$

$$= -0.96 - j1.28 = 1.6\angle -126.9^\circ$$

The Norton impedance is obtained by eliminating the voltage source on the right:

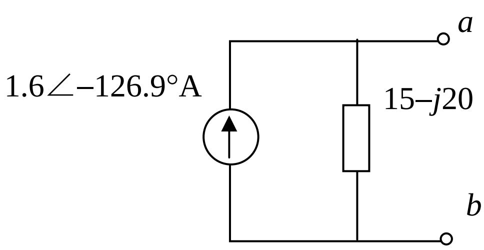


Hence,

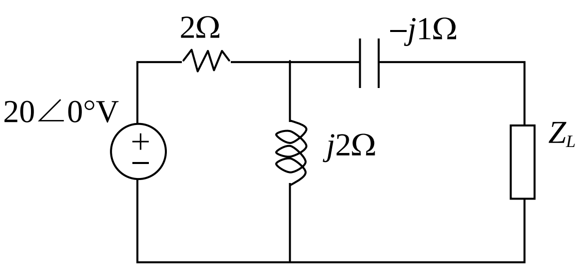
$$Z_N = 15 + \frac{1}{\dfrac{1}{j20} + \dfrac{1}{-j10}}$$

$$= 15 - j20 = 25\angle -53.1^\circ$$

The equivalent circuit is

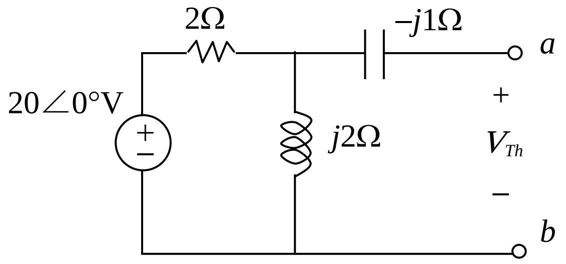


3. Find Z_L that will absorb the maximum power and determine the maximum power. (20%)



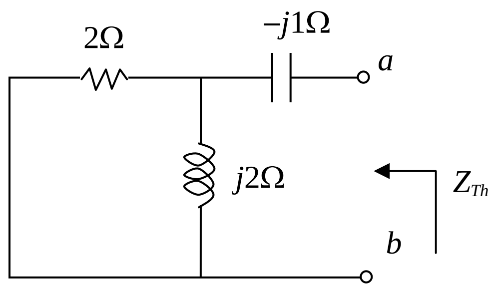
Solution:

First, find the Thevenin circuit. The Thevenin voltage V_{Th} is the voltage across the open terminals *a* to *b* as below:



It can be found that $V_{Th}=20\times\frac{j2}{2+j2}=10+j10=14.14\angle 45°$

The Thevenin impedance is obtained by eliminating the voltage source as below:



Hence,

$$Z_{Th}=-j+\frac{1}{\frac{1}{2}+\frac{1}{j2}}=1$$

Clearly, $Z_L=Z_{Th}^{*}=1\Omega$ will absorb the maximum power and the maximum power is

$$P_L=\frac{V_{Th}^2}{8R_L}=\frac{14.14^2}{8}=25\text{ W}$$

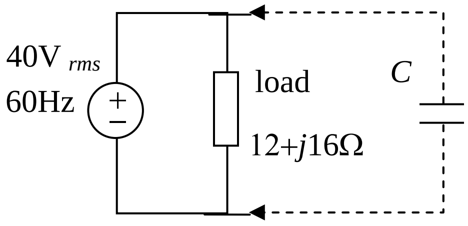
4. In the circuit on the right, find (20%)

(A) the average power dissipated in the load,

(B) the reactive power delivered by the source,

(C) the power factor,

(D) the capacitance *C* that will give a unity power factor when connected to the load.



Solution:

The effective current through the load is

$$I_L=\frac{40}{12+j16}=1.2-j1.6$$

The complex power is $S_L=V_L I_L^{*}=40(1.2+j1.6)=48+j64\text{ VA}$.

(A) The average power is 48 W.

(B) The reactive power is 64 VAR

(C) The power factor is $pf = \dfrac{48}{\sqrt{48^2+64^2}} = 0.6$

(D) The admittance of the load is $\dfrac{1}{12+j16} = \dfrac{12-j16}{400}$.

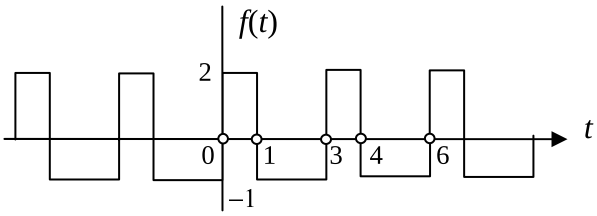
To get a unity power factor, the capacitance must satisfy $j\omega C = j\dfrac{16}{400}$, i.e.,

$$C = \frac{16}{400\omega} = \frac{16}{400\times 2\pi\times 60} = 106\mu\text{F}$$

5. The periodic function $f(t)$ can be expressed as the following Fourier series:. (20%)

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n cos(n\omega_0 t) + b_n sin(n\omega_0 t)\right)$$

Find a_k, k=0,1,2,3,4, and b_k, k =1,2,3,4,5.



Solution:

The Fourier series expansion is

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n cos(n\omega_0 t) + b_n sin(n\omega_0 t)\right)$$

where $\omega_0 = \dfrac{2\pi}{T} = \dfrac{2\pi}{3}$ and $a_0 = \dfrac{1}{3}\left[\int_0^1 2\,dt + \int_1^3 (-1)dt\right] = 0$

$$a_n = \frac{2}{3}\left[\int_0^1 2cos(n\omega_0 t)dt - \int_1^3 cos(n\omega_0 t)dt\right]$$
$$= \frac{2}{3}\left[\left.\frac{2sin(n\omega_0 t)}{n\omega_0}\right|_0^1 - \left.\frac{sin(n\omega_0 t)}{n\omega_0}\right|_1^3\right] = \frac{3}{n\pi}sin\left(\frac{2n\pi}{3}\right)$$

$$b_n = \frac{2}{3}\left[\int_0^1 2sin(n\omega_0 t)dt - \int_1^3 sin(n\omega_0 t)dt\right]$$
$$= \frac{2}{3}\left[-\left.\frac{2cos(n\omega_0 t)}{n\omega_0}\right|_0^1 + \left.\frac{cos(n\omega_0 t)}{n\omega_0}\right|_1^3\right] = \frac{3}{n\pi}\left(1-cos\left(\frac{2n\pi}{3}\right)\right)$$

Hence, $a_0 = 0$, $a_1 = \dfrac{3\sqrt{3}}{2\pi}$, $a_2 = -\dfrac{3\sqrt{3}}{4\pi}$, $a_3 = 0$, $a_4 = \dfrac{3\sqrt{3}}{8\pi}$

$b_1 = \dfrac{9}{2\pi}$, $b_2 = \dfrac{9}{4\pi}$, $b_3 = 0$, $b_4 = \dfrac{9}{8\pi}$, $b_5 = \dfrac{9}{10\pi}$