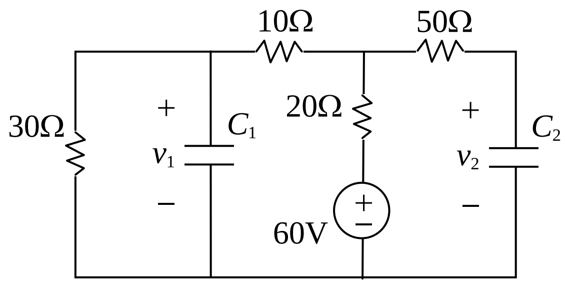
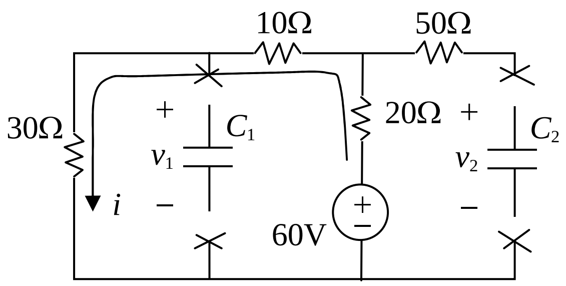
1. Find v_1 and v_2 in the circuit under dc conditions.



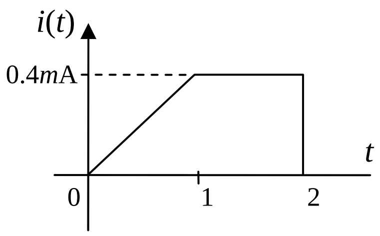
Solution:

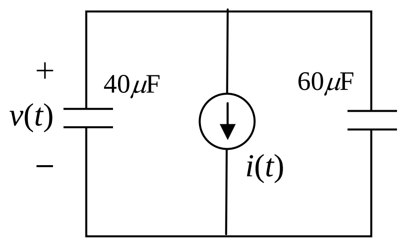
Under dc conditions, all the capacitors are treated as open and thus, the circuit is redrawn as



The current i is $i=\dfrac{60}{20+10+30}=1\,\text{A}$. Hence, $v_1=30i=30\,\text{V}$ and $v_2=(10+30)i=40\,\text{V}$.

2. Given the following circuit, sketch $v(t)$ for t>0 if $v(0)$=1.



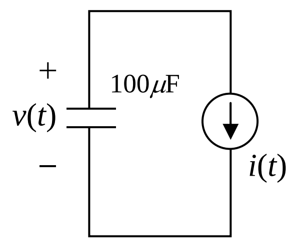


Solution:

The circuit is equivalent to the circuit on the right.

Hence,

$$v(t)=v(t_0)-\frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau = v(t_0)-10^4\int_{t_0}^{t} i(\tau)d\tau$$



For 0<t<1, $i(t)=4\times10^{-4}\,t\,\text{A}$

and then $v(t)=1-10^4\int_0^t (4\times10^{-4}\tau)d\tau = 1-2t^2$ V.

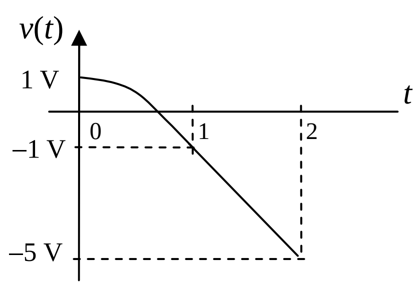
At t=1, $v(1)=-1$ V.

For 1<t<2, $i(t)=4\times10^{-4}$ A and then

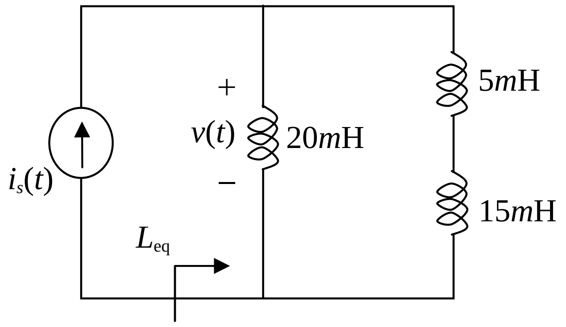
$v(t)=-1-10^4\int_1^t (4\times10^{-4})d\tau=-1-4(t-1)$ V.

At t=2, $v(2)=-5$ V.

The curve is shown on the right.



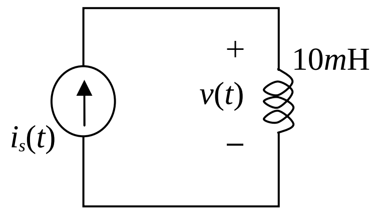
3. In the following circuit, if $i_s(t)=5e^{-2t}$ mA, find L_{eq} and $v(t)$.



Solution:

The equivalent inductance is $L_{eq}=\dfrac{1}{\dfrac{1}{20}+\dfrac{1}{5+15}}=10\,m\,\mathrm{H}$.

The circuit is equivalent to the circuit on the right.



Hence, $v(t)=L_{eq}\dfrac{di_s(t)}{dt}=10^{-2}\times\left(-10e^{-2t}\right)=-0.1e^{-2t}$ mH.

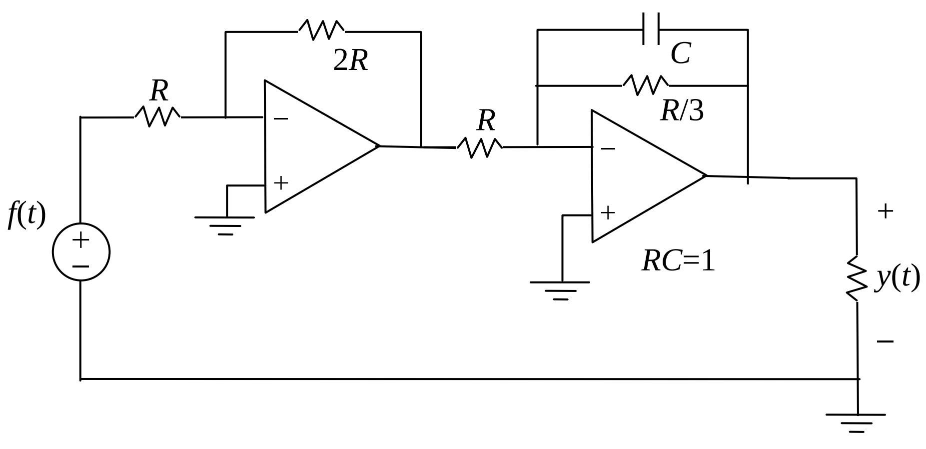
4. Design an analog computer circuit to solve the following ordinary differential equation

$$\frac{dy(t)}{dt}+3y(t)=2f(t)$$

where $f(t)$ is the input and $y(t)$ is the output. Neglect the initial condition in this problem.

Solution:

The ODE is rewritten as $\dfrac{dy(t)}{dt}=-3y(t)+2f(t)$ and the circuit is designed as below:



5. Express the following signal in terms of singularity functions.

$$x(t)=\begin{cases} t & 0<t<2 \\ 3 & 2<t<3 \\ 6-t & 3<t<8 \\ 0 & \text{otherwise} \end{cases}$$

Solution:

The signal can be expressed as

$$x(t)=r(t)-r(t-2)+u(t-2)-r(t-3)+r(t-8)+2u(t-8)$$

6. Evaluate the following integrals:

(A) $\int_{-\infty}^{2} e^{(t-3)/2}\,\delta(t-1)dt$

(B) $\int_{-2}^{2} (cos(3t\pi)\delta(t+3)+cos(2t\pi)\delta(t-1))dt$

(C) $\int_{-\infty}^{\infty} 5(t-7)^7\,\delta(6-t)dt$

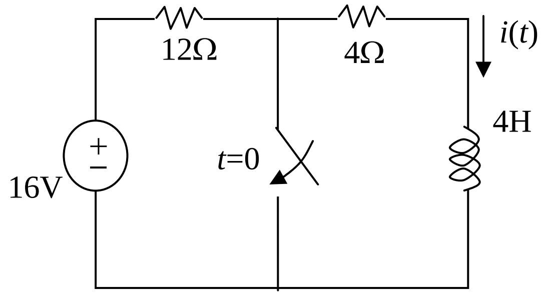
Solution:

(A) $\int_{-\infty}^{2} e^{(t-3)/2}\,\delta(t-1)dt = e^{-1} = 0.368$

(B) $\int_{-2}^{2} (cos(3t\pi)\delta(t+3)+cos(2t\pi)\delta(t-1))dt = cos(2\pi) = 1$

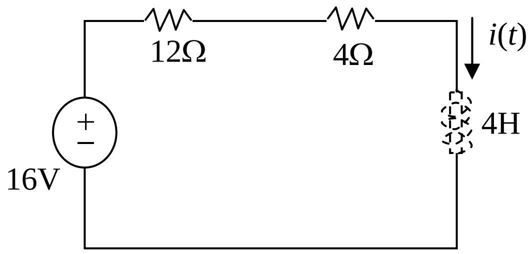
(C) $\int_{-\infty}^{\infty} 5(t-7)^7\,\delta(6-t)dt = -5$

7. The circuit has been operated for a long time and the switch is closed at t=0. Determine $i(t)$ for t>0.



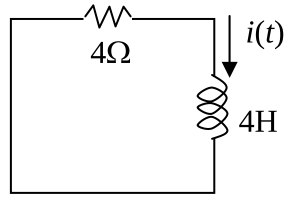
Solution:

For $t=0^-$, we have the circuit on the right(up).



The current $i(0^-)=\dfrac{16}{12+4}=1\,\text{A}$.

Since the inductor current can not abruptly change, at t=0 we have i(0)=1 A. Now for t>0, the circuit is shown on the right(down) and it is described by

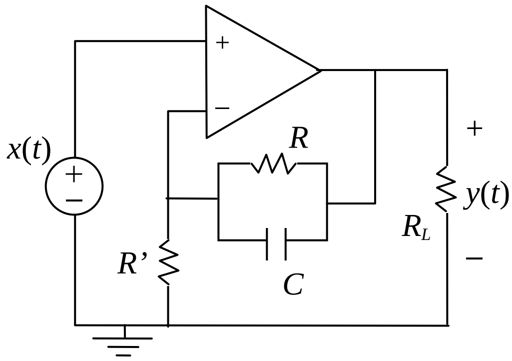


$$\frac{di(t)}{dt}+i(t)=0, \qquad i(0)=1\,\text{A}$$

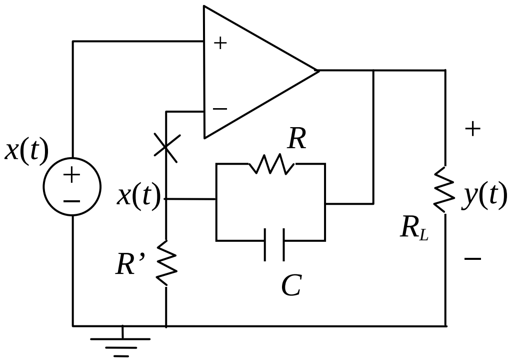
where R/L=1. Hence, the solution is

$i(t)=e^{-t}$ A for t>0.

8. Write the ordinary differential equation to describe the relationship between the output *y*(*t*) and the input *x*(*t*) of the following circuit which contains an ideal op amp. Note that *R*, *R'*, R_L and *C* are given and the initial condition *y*(0) is set to be 0.



Solution:



Since the use of an ideal op amp, we have the voltage *x*(*t*) at the '−' terminal, which is open. Hence, according to KCL, the circuit satisfies

$$\frac{x(t)}{R'} = \frac{y(t) - x(t)}{R} + C\frac{d(y(t) - x(t))}{dt}, \qquad y(0) = 0 \text{ V}$$

$$\Rightarrow \dot{y}(t) + \frac{1}{RC}y(t) = \dot{x}(t) + \left(\frac{1}{RC} + \frac{1}{R'C}\right)x(t), \qquad y(0) = 0 \text{ V}$$