1. In electric circuit, there are four basic *SI* units, m, kg, s, and A. Based on these basic units, some derived units are often used for physical quantities, such as W(watt), C(coulomb) and Ω(ohm) and V(volt) respectively for electric power, charge, resistance and voltage. Please write the *SI* units for the above derived units: W, C and Ω.

Solution:

$$\mathrm{W} := \frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}^3}, \mathrm{C} := \mathrm{A}\cdot\mathrm{s}, \quad \Omega := \frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}^3\cdot\mathrm{A}^2}$$

2. (A) Determine the current at t=0.1 s, $i_1(0.1)$, through an element if the flowing charge is $q_1(t) = e^{-2t}\, sin(4t)\,\mu\mathrm{C}$.

   (B) Find the charge Q_2 flowing through a device from t=0 to t=0.1 s if the current is $i_2(t) = e^{-2t}\, sin(4t)\, m\mathrm{A}$. [Hint: $\int e^{-at}\, sin(bt)dt = e^{-at}(\alpha\, sin(bt) + \beta\, cos(bt))$

Solution:

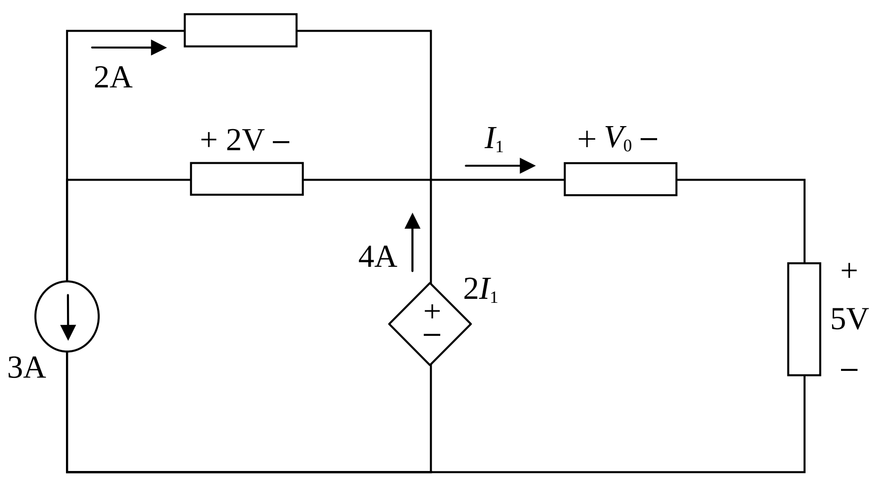
(A) The current is $i_1(t) = \dfrac{dq_1(t)}{dt} = -2e^{-2t}\, sin(4t) + 4e^{-2t} cos(4t)$, and thus

$$i_1(0.1) = -2e^{-0.2}\, sin(0.4) + 4e^{-0.2} cos(0.4) = 2.3787\,\mu\mathrm{A}$$

(B) The charge is $q_2(t) = \int e^{-2t}\, sin(4t)dt = -0.1e^{-2t}\, sin(4t) - 0.2e^{-2t}\, cos(4t)$, and thus

$$Q_2 = \int_0^{0.1} e^{-2\tau}\, sin(4\tau)d\,\tau = -0.1e^{-2t}\, sin(4t) - 0.2e^{-2t}\, cos(4t)\Big|_0^{0.1} = 0.0173\, m\mathrm{C}$$

3. Find V_0 and the power absorbed by the current-controlled voltage source.



Solution:

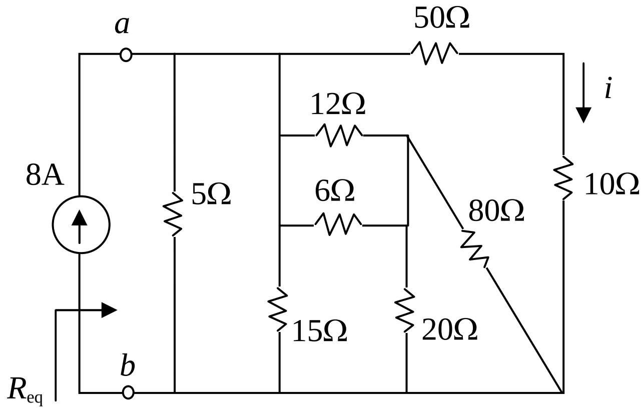
It is easy to find that $3+I_1=4$, and thus $I_1=1$.
Besides, according to KVL, we obtain $V_0+5=2I_1=2$, and thus $V_0=-3\text{ V}$.
Based on passive sign convention, the power absorbed
by the current-controlled voltage source is $(2I_1)\times(-4)=-8\text{ W}$

4. A 100-W incandescent bulb operates 2 hours a day. What is the money to pay for its operation in 30 days if the electricity costs 2.5 dollars/*k*Wh.

Solution:

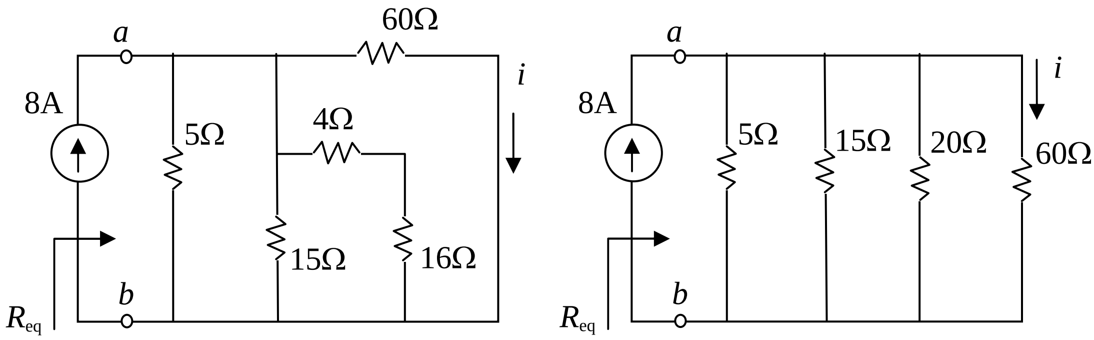
The total electric energy is $E=100\times2\times30=6000\text{ Wh}=6\,k\text{ Wh}$,
and thus the money to pay for its operation is $2.5\times6=15\text{ dollars}$.

5. (A) Find the equivalent resistance R_{eq} at terminals *a*-*b*.
   (B) Determine the current *i* in the circuit.



Solution:

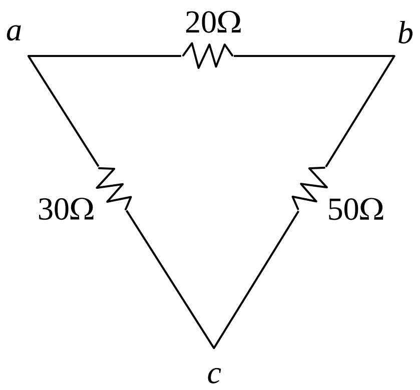
(A) The circuit can be rearranged as below:



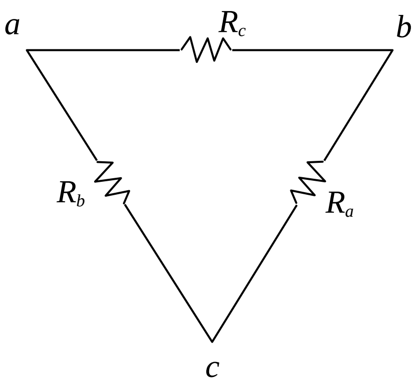
Hence, $R_{\mathrm{eq}} = \left(\frac{1}{5}+\frac{1}{15}+\frac{1}{20}+\frac{1}{60}\right)^{-1} = 3\ \ \Omega.$

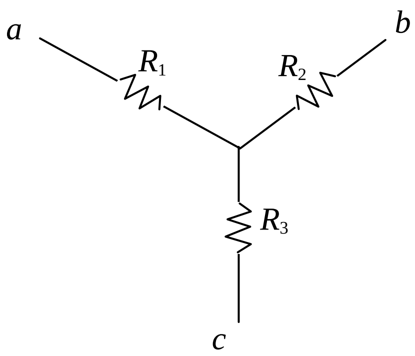
(B) The current is $i = \dfrac{\frac{1}{60}}{\frac{1}{5}+\frac{1}{15}+\frac{1}{20}+\frac{1}{60}} \times 8 = \frac{1}{20} \times 8 = 0.4\ \ \text{A}.$

6. Transform the circuit from Δ to Y.



Solution:





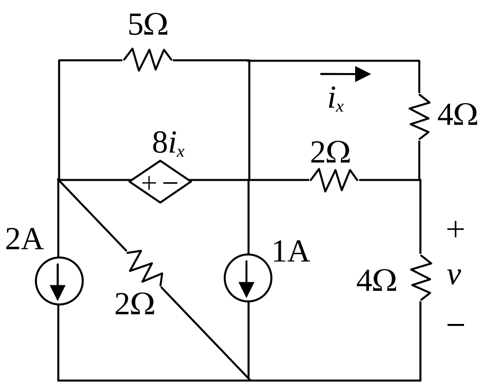
According to the transformation formula, we have

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{600}{50+30+20} = 6\ \ \Omega$$

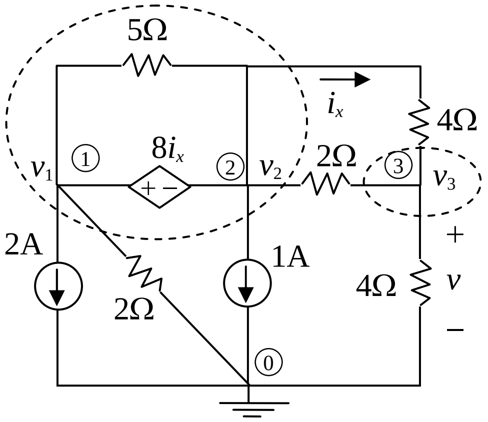
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{1000}{50+30+20} = 10\ \ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{1500}{50+30+20} = 15\ \ \Omega$$

7. Based on the node-voltage method, write a set of node voltage equations and determine the voltage v in the circuit.



Solution:



First, choose the reference point and number the node voltages v_1, v_2 and v_3.

There is a supernode containing ① and ②. The node voltage equations are

Supernode①②:

$$\text{KVL:}\quad v_1 - v_2 = 8i_x = 8\left(\frac{v_2 - v_3}{4}\right) = 2v_2 - 2v_3 \Rightarrow v_1 - 3v_2 + 2v_3 = 0$$

$$\text{KCL:}\quad \frac{v_1}{2} + \frac{v_2 - v_3}{2} + \frac{v_2 - v_3}{4} = -2 - 1 \Rightarrow 2v_1 + 3v_2 - 3v_3 = -12$$

KCL③:

$$\frac{v_3}{4} + \frac{v_3 - v_2}{2} + \frac{v_3 - v_2}{4} = 0 \Rightarrow 4v_3 - 3v_2 = 0$$

The node voltage equations can be solved as $v_1 = -4.8$ V, $v_2 = -3.2$ V and $v_3 = -2.4$ V. Hence, the voltage $v = v_3 = -2.4$ V.