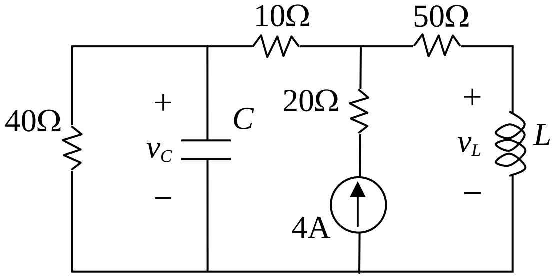
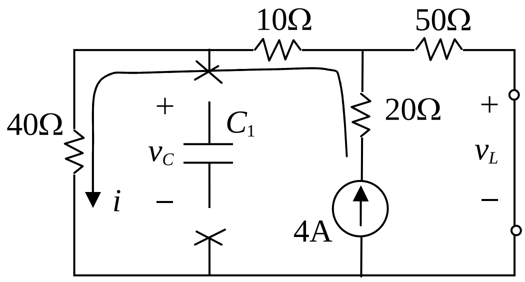
1. Find v_C and v_L in the circuit under dc conditions.



Solution:

Under dc condition, C is open and L is short. Thus, the circuit is redrawn as



The current i is $i = \frac{50}{50+(10+40)} \times 4 = 2\text{ A}$. Hence, the voltage of C is $v_C = 40i = 80\text{ V}$. Since the inductor L is short, its voltage $v_L = 0\text{ V}$.

2. Express the following signal in terms of singularity functions.

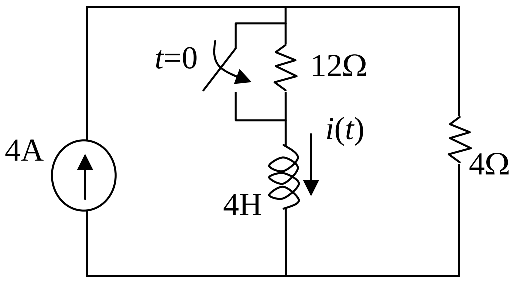
$$x(t) = \begin{cases} 1 & -1 < t < 1 \\ 3 & 2 < t < 3 \\ 2 & 3 < t < 7 \\ 0 & \text{otherwise} \end{cases}$$

Solution:

The signal can be expressed as

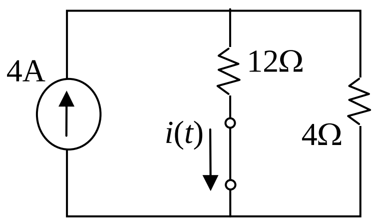
$$x(t) = u(t+1) - u(t-1) + 3u(t-2) - u(t-3) - 2u(t-7)$$

3. The switch has been operated for a long time and then it is closed at t=0. Determine $i(t)$ for t>0.



Solution:

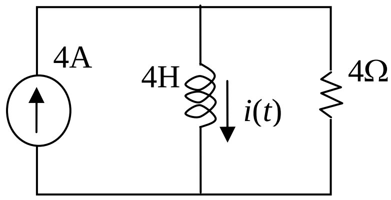
For $t=0^-$, the circuit is on the right and $i(0^-) = 4 \times \frac{4}{12+4} = 1\text{ A}$. Since the inductor current can not abruptly change, at t=0 we have $i(0)$=1 A.



Now for t>0, the circuit is redrawn on the right, which is described by

$$\frac{di(t)}{dt}+i(t)=4\,,\quad i(0)=1\,\text{A}$$

where L/R=1. Hence, the solution is $i(t)=-3e^{-t}+4\,\text{A}$ for t>0.



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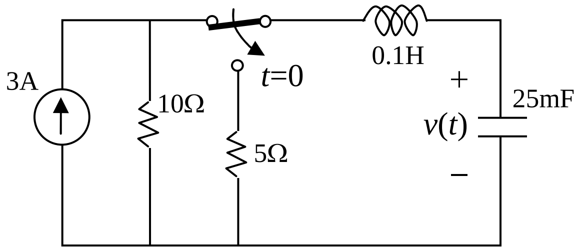
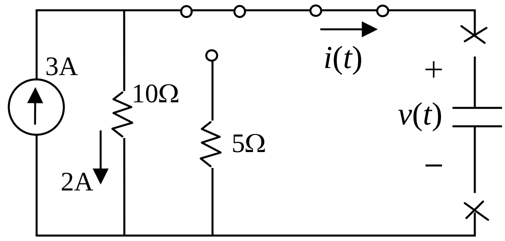
4. The following circuit has been operated for a long time before t=0, find $v(t)$ for t>0.

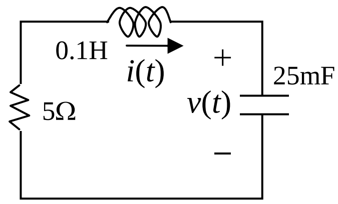


Solution:

For $t=0^-$, the circuit is redrawn on the right, where the inductor is short and the capacitor is open. Hence, $v(0^-)=3\times 10=30\,\text{V}$ and $i(0^-)=0$.



Then we have $\dot{v}(0^-)=\dfrac{1}{0.04}\times i(0^-)=0$.



For t>0, the circuit is redrawn on the right without source. and then $\ddot{v}(t)+50\dot{v}(t)+400v(t)=0$, $v(0)=30$, $\dot{v}(0)=0$.

The characteristic equation is $\lambda^2+50\lambda+400=0$ and the eigenvalues are $\lambda=-10$ or -40. The solution only contains homogeneous part, which is written as

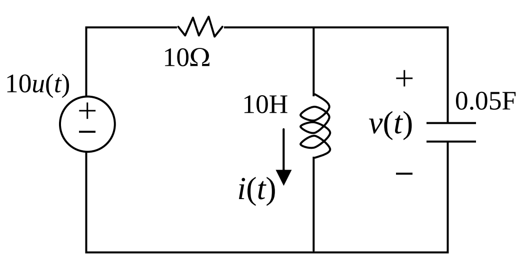
$$v(t)=A_1e^{-10t}+A_2e^{-40t}$$

Its derivative is $\dot{v}(t)=-10\,A_1e^{-10t}-40\,A_2e^{-40t}$. Therefore, from initial conditions we have

$$v(0)=A_1+A_2=30 \qquad \text{and} \qquad \dot{v}(0)=-10\,A_1-40\,A_2=0$$

which result in $A_1=40$ and $A_2=-10$. Therefore, $v(t)=40e^{-10t}-10e^{-40t}$.

5. In the following circuit, find $v(t)$ for t>0. Assume $v(0)$=0 V and $i(0)$=2 A.

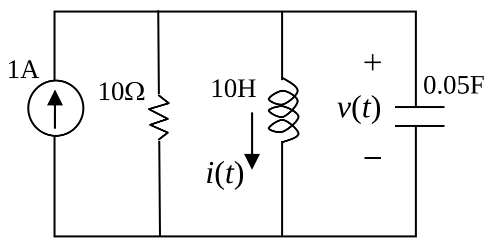


Solution:

Since $v(t)=L\dfrac{di(t)}{dt}$,

we have $\dfrac{di(0^+)}{dt}=\dfrac{1}{L}v(0^+)=\dfrac{1}{L}v(0)=0$.

For t>0, the circuit is redrawn on the right,



which is described as

$$\frac{d^2i(t)}{dt^2}+\frac{1}{RC}\frac{di(t)}{dt}+\frac{i(t)}{LC}=\frac{1}{LC}\Rightarrow\frac{d^2i(t)}{dt^2}+2\frac{di(t)}{dt}+2\,i(t)=2$$

with initial conditions $\frac{di(0^+)}{dt}=0$ and $i(0^+)= i(0)=2$.

The characteristic equation is $\lambda^2+2\lambda+2=0$

and the eigenvalues are $\lambda=-1+j$ or $-1-j$. The solution becomes

$$i(t)=e^{-t}\left(A_1cos(t)+A_2sin(t)\right)+1$$

Its derivative is $\frac{di(t)}{dt}=-e^{-t}\left(A_1cos(t)+A_2sin(t)\right)+e^{-t}\left(-A_1sin(t)+A_2cos(t)\right)$. Therefore, from initial conditions we have

$$i(0^+)=A_1+1=2 \qquad \text{and} \qquad \frac{di(0^+)}{dt}=-A_1+A_2=0$$

which result in $A_1=1$ and $A_2=1$. Therefore, $i(t)=e^{-t}\left(cos(t)+sin(t)\right)+1$, which leads to

$$v(t)=10\frac{di(t)}{dt}=-20e^{-t}sin(t)$$

6. If $v_1(t)=10cos(40t+\pi/4)$, $v_2(t)=15cos(40t+30°)$ and $v(t)=v_1(t)+v_2(t)$, find the amplitude, phase and frequency f of $v(t)$.

Solution:

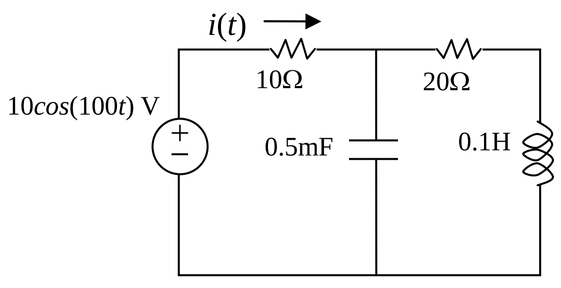
The phasors of $v_1(t)$ and $v_2(t)$ $\boldsymbol{V}_1=10\angle 45°$ and $\boldsymbol{V}_2=15\angle 30°$.

Then the phasor of $v(t)$ is

$$\begin{aligned}\boldsymbol{V}=\boldsymbol{V}_1+\boldsymbol{V}_2&=10\angle 45°+15\angle 30°\\&=10\left(\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right)+15\left(\frac{\sqrt{3}}{2}+j\frac{1}{2}\right)\\&=10(0.707+j0.707)+15(0.866+j0.5)\\&=7.07+j7.07+12.99+j7.5\\&=20.06+j14.57=24.7929\angle 35.99°\end{aligned}$$

The sum is $v(t)=24.7929cos(40t+35.99^0)$. Thus, its amplitude is 24.7929, its phase is 35.99^{o}, and the frequency is $f=\frac{40}{2\pi}=6.3662\,s^{-1}$.

7. In the following circuit, determine the steady-state current $i(t)$.

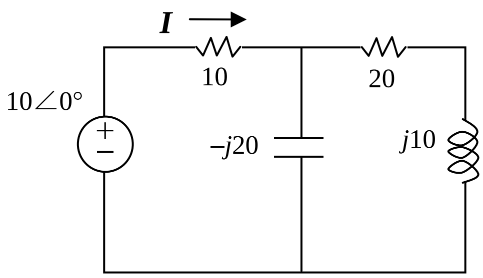


Solution:

Represent the circuit in the form of phasor on the right. The current phasor is obtained as

$$\boldsymbol{I} = \frac{10\angle 0^\circ}{10 + \frac{-j20(20+j10)}{(-j20)+(20+j10)}}$$

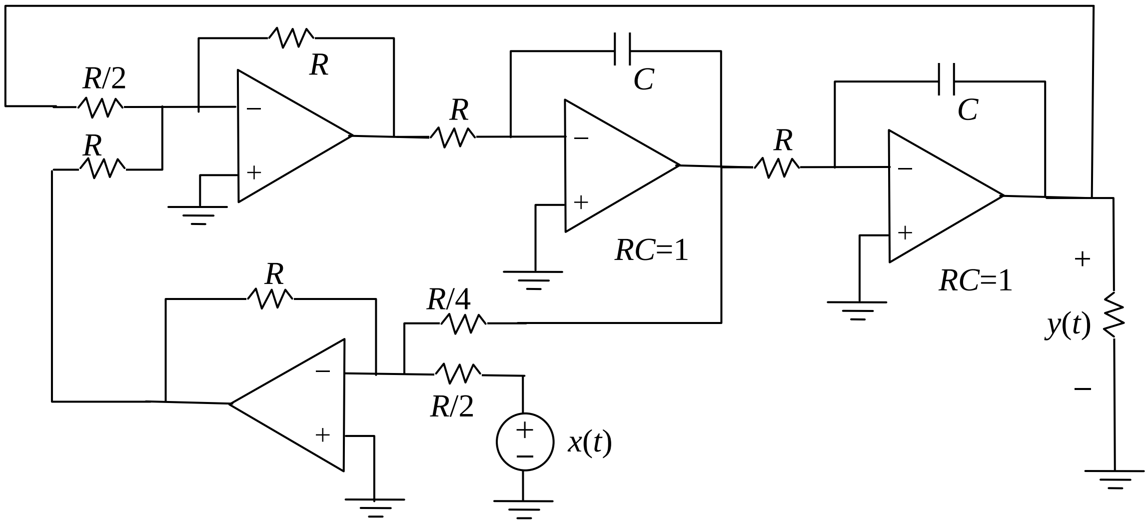
$$= \frac{13+j6}{41} = 0.3492\angle 24.7747^\circ$$



Hence,

$$i(t) = 0.3492\cos(100t + 24.7747^\circ)$$

8. The following analog computer circuit is used to solve an ordinary differential equation (ODE) with input $x(t)$ and output $y(t)$. If the initial conditions are neglected, what is the ODE?



Solution:

The ODE is $\frac{d^2y(t)}{dt^2} = -4\frac{dy(t)}{dt} - 2y(t) + 2x(t)$.