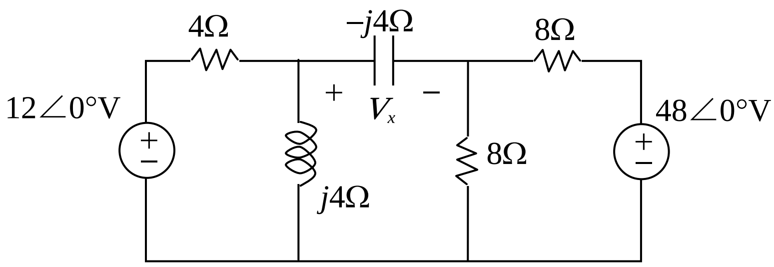
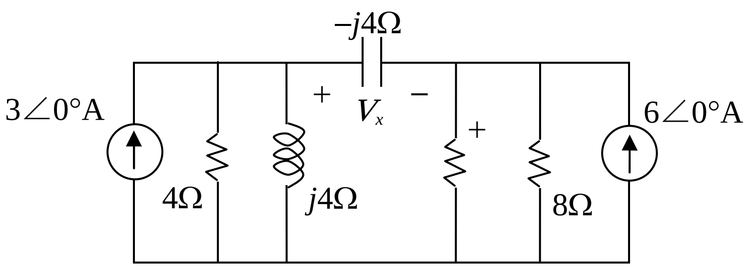
1. Determine voltage V_x in the following circuit. (15%)

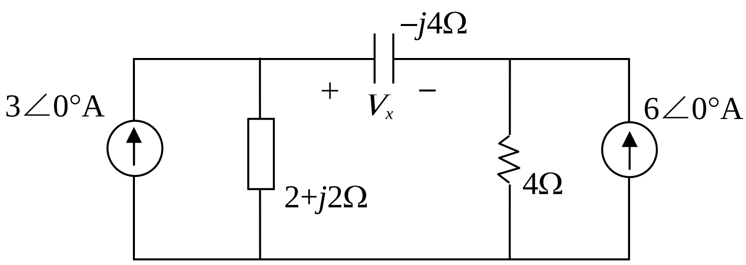


Solution:

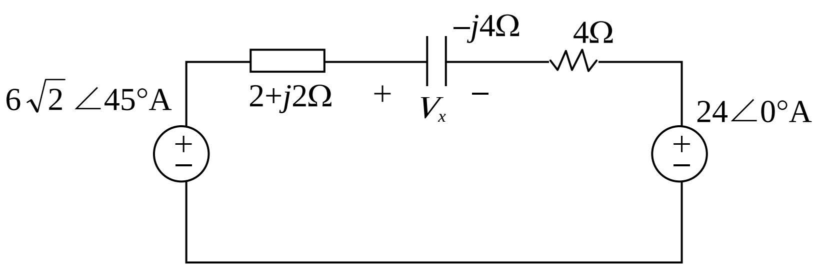
Use source transformation, the circuit is redrawn as below:



Further, it is changed as below:



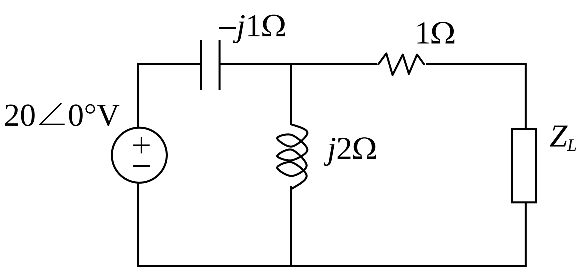
and then



Hence,

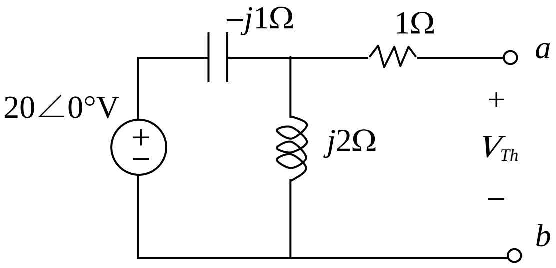
$$V_x = \frac{-j4}{2+j2-j4+4}\left(6\sqrt{2}\angle 45^\circ - 24\angle 0^\circ\right) = j12\text{ A} = 12\angle 90^\circ\text{ A}$$

2. Find Z_L that will absorb the maximum power and determine the maximum power. (10%)



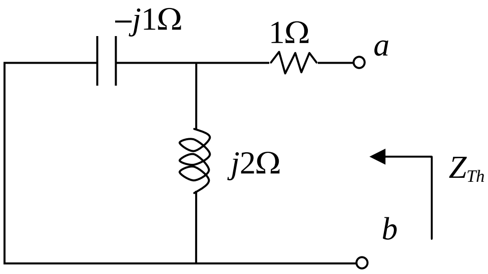
Solution:

First, find the Thevenin circuit. The Thevenin voltage V_{Th} is the voltage across the open terminals a to b as below:



It can be found that $V_{Th} = 20 \times \dfrac{j2}{-j1 + j2} = 40 = 40\angle 0^\circ$

The Thevenin impedance is obtained by eliminating the voltage source as below:



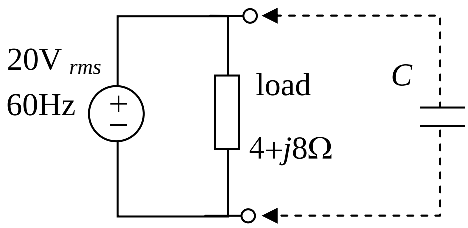
Hence,

$$Z_{Th} = 1 + \frac{1}{\dfrac{1}{-j1} + \dfrac{1}{j2}} = 1 - j2$$

Clearly, $Z_L = R_L + jX_L = Z_{Th}^* = 1 + j2\,\Omega$ will absorb the maximum power and the maximum power is

$$P_L = \frac{V_{Th}^2}{8R_L} = \frac{40^2}{8} = 200 \text{ W}$$

3. Consider the circuit on the right. First, if the capacitor is not connected, find (12%)



(A) the average power dissipated in the load,

(B) the reactive power delivered by the source,

(C) the power factor.

Then, connect the capacitor to the circuit and adjust the power factor to 1.

(D) What is the capacitance C?

Solution:

When the capacitor is not connected to the circuit, the effective current through the load is

$$I_L = \frac{20}{4+j8} = 1 - j2$$

The complex power is $S_L = V_L I_L^* = 20(1+j2) = 20 + j40$ VA .

(A) The average power is 20 W.

(B) The reactive power is 40 VAR

(C) The power factor is $pf = \frac{20}{\sqrt{20^2+40^2}} = \frac{1}{\sqrt{5}} = 0.4472$

(D) The admittance of the load is $\frac{1}{4+j8} = \frac{1-j2}{20}$.

To get a unity power factor, the capacitance must satisfy $j\omega C = j\frac{2}{20}$, i.e.,

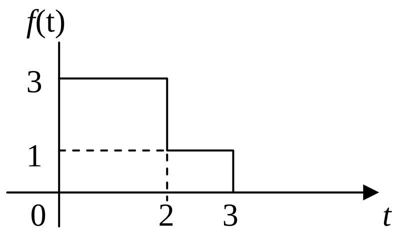
$$C = \frac{2}{20\omega} = \frac{2}{20\times 2\pi \times 60} = 265\mu\text{F}$$

4. Find the Laplace transform of the following signals: (15%)

(A) $f(t)$ shown on the right.

(B) $g(t) = \left((t+2)e^{-t}\cos 2t\right)u(t)$

(C) $h(t) = \left(t^2 + 2t + 2 - e^{-t}\cos 2t\right)\delta(t)$



Solution:

(A) $F(s) = \int_{0^-}^{\infty} f(t)e^{-st}dt = \int_0^2 3e^{-st}dt + \int_2^3 e^{-st}dt = \frac{3}{s} - \frac{2}{s}e^{-2s} - \frac{1}{s}e^{-3s}$

(B) $G(s) = \int_{0^-}^{\infty} g(t)e^{-st}dt = 2\int_0^{\infty} e^{-t}\cos(2t)e^{-st}dt + \int_0^{\infty} t\cdot e^{-t}\cos(2t)e^{-st}dt$

$$= 2\frac{s+1}{(s+1)^2+4} - \frac{d}{ds}\left[\frac{s+1}{(s+1)^2+4}\right] = \frac{2(s+1)}{(s+1)^2+4} + \frac{(s+1)^2-4}{\left((s+1)^2+4\right)^2}$$

$$= \frac{2(s+1)^3 + (s+1)^2 + 8(s+1) - 4}{\left((s+1)^2+4\right)^2}$$

(C) Since $h(t) = \left(t^2 + 2t + 2 - e^{-t}\cos 2t\right)\delta(t) = \delta(t)$, we have $H(s)$=1.

5. Determine the inverse Laplace transform of the following signals: (15%)

(A) $F(s) = \frac{2s+3}{(s+1)(s+3)(s+5)}$

(B) $G(s) = \frac{s^2+5}{(s+2)^2(s+4)}$

(C) $H(s)=\dfrac{s+6}{(s+1)(s^2+4s+8)}$

Solution:

(A) Since $F(s)=\dfrac{2s+3}{(s+1)(s+3)(s+5)}=\dfrac{1/8}{s+1}+\dfrac{3/4}{s+3}+\dfrac{-7/8}{s+5}$,

we have $f(t)=\dfrac{1}{8}e^{-t}+\dfrac{3}{4}e^{-3t}-\dfrac{7}{8}e^{-5t}$, t>0.

(B) Since $G(s)=\dfrac{s^2+5}{(s+2)^2(s+4)}=\dfrac{21/4}{s+4}+\dfrac{-17/4}{s+2}+\dfrac{9/2}{(s+2)^2}$,

we have $g(t)=\dfrac{21}{4}e^{-4t}-\dfrac{17}{4}e^{-2t}+\dfrac{9}{2}te^{-2t}$, t>0.

(C) Since $H(s)=\dfrac{s+6}{(s+1)(s^2+4s+8)}=\dfrac{1}{s+1}-\dfrac{s+2}{(s+2)^2+2^2}$,

we have $h(t)=e^{-t}-e^{-2t}cos\,2t$, t>0.

6. Let $F(s)=\dfrac{s^2+3}{s^3-4s^2+2s+1}$ and determine the initial and final values of $f(t)$. (6%)

Solution:

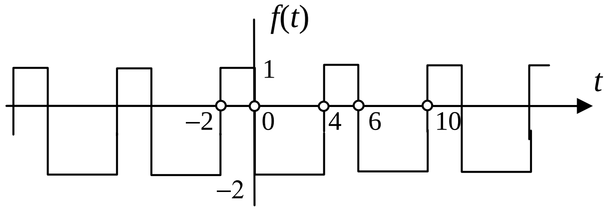
From initial value theorem, we have $f(0)=\lim\limits_{s\to\infty} sF(s)=\lim\limits_{s\to\infty}\dfrac{s(s^2+3)}{s^3-4s^2+2s+1}=\lim\limits_{s\to\infty}\dfrac{s^3}{s^3}=1$.

For the truth of finial value theorem $f(\infty)=\lim\limits_{s\to 0} sF(s)$, it is required that $F(s)$ is stable, i.e., all its poles are located on the left-half plane, otherwise $f(\infty)=\infty$. Since $F(s)$ has a ploe at s=1, not on the left-half plane, we know that $f(\infty)=\infty$.

7. The periodic function $f(t)$ can be expressed as the following Fourier series:. (12%)

$$f(t)=a_0+\sum_{n=1}^{\infty}\left(a_n cos(n\omega_0 t)+b_n sin(n\omega_0 t)\right)$$

Find a_k, k=0,1,2,3,4, and b_k, k =1,2,3,4,5.



Solution:

The Fourier series expansion is

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n cos(n\omega_0\, t) + b_n sin(n\omega_0\, t)\right)$$

where $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}$ and $a_0 = \frac{1}{6}\left[\int_{-2}^{0} dt + \int_{0}^{4} (-2)dt\right] = -1$

$$a_n = \frac{2}{6}\left[\int_{-2}^{0} cos(n\omega_0\, t)dt + \int_{0}^{4}(-2)cos(n\omega_0\, t)dt\right]$$

$$= \frac{1}{3}\left[\left.\frac{sin(n\omega_0\, t)}{n\omega_0}\right|_{-2}^{0} - 2\left.\frac{sin(n\omega_0\, t)}{n\omega_0}\right|_{0}^{4}\right] = \frac{3\,sin(2n\pi/3)}{n\pi}$$

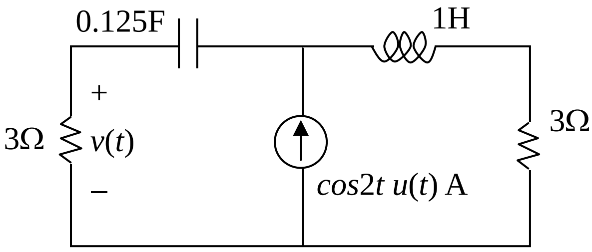
$$b_n = \frac{2}{6}\left[\int_{-2}^{0} sin(n\omega_0\, t)dt + \int_{0}^{4}(-2)sin(n\omega_0\, t)dt\right]$$

$$= \frac{1}{3}\left[-\left.\frac{cos(n\omega_0\, t)}{n\omega_0}\right|_{-2}^{0} + 2\left.\frac{cos(n\omega_0\, t)}{n\omega_0}\right|_{0}^{4}\right] = -\frac{3}{n\pi} + \frac{3cos(2n\pi/3)}{n\pi}$$

Hence, $a_0 = 1$, $a_1 = \frac{3\sqrt{3}}{2\pi}$, $a_2 = \frac{-3\sqrt{3}}{4\pi}$, $a_3 = 0$, $a_4 = \frac{3\sqrt{3}}{8\pi}$

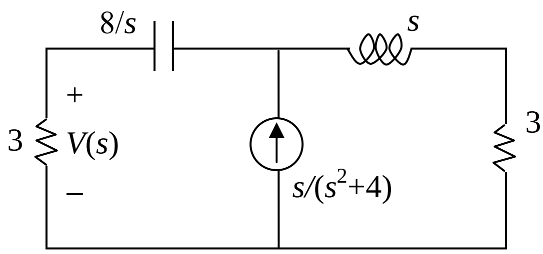
$b_1 = -\frac{9}{2\pi}$, $b_2 = -\frac{9}{4\pi}$, $b_3 = 0$, $b_4 = -\frac{9}{8\pi}$, $b_5 = -\frac{9}{10\pi}$

8. Determine $v(t)$. (15%)



Solution:

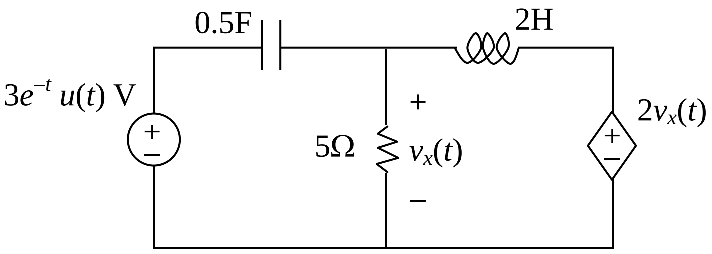
The circuit is redrawn as below:



$$V(s) = 3\times\frac{s}{s^2+4}\times\frac{s+3}{s+3+3+8/s} = \frac{3s^2(s+3)}{(s^2+4)(s^2+6s+8)} = \frac{1.05s-2.7}{s^2+4} + \frac{0.25}{s+2} + \frac{1.2}{s+4}$$

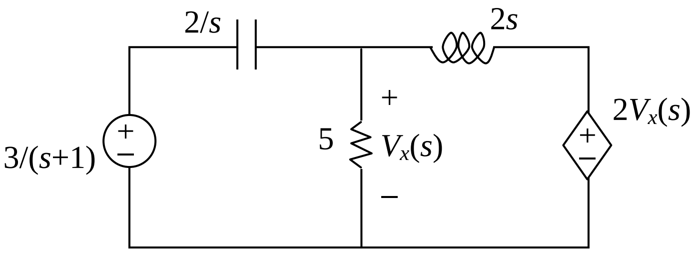
Hence, $v(t) = 1.05\,cos\,2t - 1.35\,sin\,2t + 0.25e^{-2t} + 1.2e^{-4t}$

9. Find $V_x(s)$. (15%)



Solution:

The circuit is redrawn as below:



$$\frac{V_x(s)}{2s}=\frac{V_x(s)}{5}+\frac{V_x(s)-\frac{3}{s+1}}{\frac{2}{s}}$$

$$\Rightarrow \frac{V_x(s)}{2s}=\frac{V_x(s)}{5}+\frac{s(s+1)V_x(s)-3s}{2(s+1)} \Rightarrow \frac{V_x(s)}{2s}=\frac{V_x(s)}{5}+\frac{sV_x(s)}{2}-\frac{3s}{2(s+1)}$$

$$\Rightarrow \left(\frac{1}{2s}-\frac{1}{5}-\frac{s}{2}\right)V_x(s)=-\frac{3s}{2(s+1)} \Rightarrow \frac{5-2s-5s^2}{10s}V_x(s)=-\frac{3s}{2(s+1)}$$

$$\Rightarrow V_x(s)=\frac{3s}{2(s+1)}\times\frac{10s}{5s^2+2s-5}=\frac{15s^2}{(s+1)(5s^2+2s-5)}=\frac{15s^2}{5s^3+7s^2-3s-5}$$