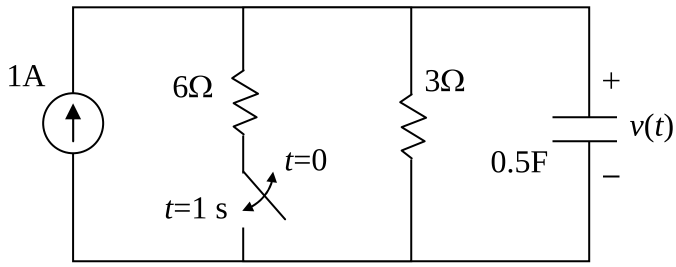
1. The switch has been closed for a long time. If the switch is opened at t=0 and is closed again at t=1 sec, determine $v(t)$ for t>0.



Sol:

Since the switch has been closed for a long time, we have $0.5\frac{dv}{dt}+\frac{v}{3}+\frac{v}{6}=1$, where $\frac{dv}{dt}\left(0^-\right)=0$.

Hence, $v\left(0^-\right)=2$ V.

For t>0, the switch is opened and then $0.5\frac{dv}{dt}+\frac{v}{3}=1 \Rightarrow \frac{dv}{dt}+\frac{2v}{3}=2$.

This leads to $v(t)=Ae^{-\frac{2}{3}t}+3$ and $v\left(0^+\right)=v\left(0^-\right)=2 \Rightarrow A+3=2 \Rightarrow A=-1$.

Hence, $v(t)=-e^{-\frac{2}{3}t}+3$ for t>0.

For t>1, the switch is closed again and then $0.5\frac{dv}{dt}+\frac{v}{3}+\frac{v}{6}=1$, where $v\left(1^-\right)=-e^{-\frac{2}{3}}+3$.

We have $0.5\frac{dv}{dt}+\frac{v}{2}=1$ and then $v(t)=Be^{-\frac{1}{2}t}+2$.

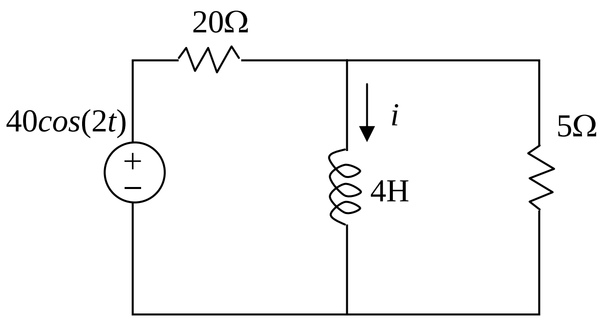
Moreover, $v\left(1^+\right)=v\left(1^-\right)=-e^{-\frac{2}{3}}+3 \Rightarrow Be^{-\frac{1}{2}}+2=-e^{-\frac{2}{3}}+3 \Rightarrow B=-e^{-\frac{1}{6}}+e^{\frac{1}{2}}=0.8022$.

This results in $v(t)=0.8022e^{-\frac{1}{2}t}+2$ V.

The total solution for t>0 is

$$v(t)=\begin{cases}-e^{-\frac{2}{3}t}+3, & 0<t<1\\ 0.8022e^{-\frac{1}{2}t}+2, & t>1\end{cases}$$

2. If $i(0^-)$=1, determine $i(t)$ for t>0.



Sol:

The equation is $i+\left(4\frac{di}{dt}\right)\frac{1}{5}+\left(4\frac{di}{dt}-40\cos(2t)\right)\frac{1}{20}=0 \Rightarrow \frac{di}{dt}+i=2\cos(2t)$.

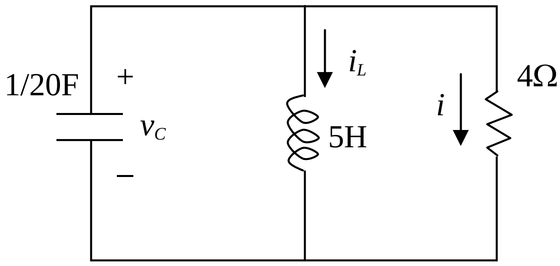
First, calculate $\left|\frac{1}{D+1}\right|_{D=j2}=\left|\frac{1}{1+j2}\right|=\frac{1}{\sqrt{5}}\angle\theta$, where $\cos\theta=\frac{1}{\sqrt{5}}$ and $\sin\theta=-\frac{2}{\sqrt{5}}$.

Hence,

$$\begin{aligned} i(t) &= Ae^{-t} + \frac{2}{\sqrt{5}}cos(2t+\theta) \\ &= Ae^{-t} + \frac{2}{\sqrt{5}}(cos(2t)cos\theta - sin(2t)sin\theta). \\ &= Ae^{-t} + \frac{2}{5}cos(2t) + \frac{4}{5}sin(2t) \end{aligned}$$

Furthermore, $i(0^+) = i(0^-) = 1 \Rightarrow A + \frac{2}{5} = 1 \Rightarrow A = \frac{3}{5}$.

The total solution is $i(t) = \frac{3}{5}e^{-t} + \frac{2}{5}cos(2t) + \frac{4}{5}sin(2t)$.



3. If $i_L(0^-)=3$, $v_C(0^-)=0$, determine $i(t)$ for $t>0$.

Sol:

The component equations are

$$C\,\dot{v}_C = i_C = -i_L - \frac{v_C}{R} \Rightarrow \dot{v}_C(0^+) = -\frac{i_L(0^+)}{C} - \frac{v_C(0^+)}{RC} = -\frac{i_L(0^-)}{C} - \frac{v_C(0^-)}{RC} = -60$$

$$L\frac{di_L}{dt} = v_L = v_C$$

The system equation is derived as $\ddot{v}_C = \frac{1}{C}\frac{di_C}{dt} = -\frac{1}{C}\frac{di_L}{dt} - \frac{1}{RC}\dot{v}_C = -\frac{1}{RC}\dot{v}_C - \frac{1}{LC}v_C$,

i.e., $\ddot{v}_C + \frac{1}{LC}v_C + \frac{1}{RC}\dot{v}_C = 0$, $v_C(0^+) = v_C(0^-) = 0$ and $\dot{v}_C(0^+) = -60$.

We have

$$\ddot{v}_C + 5\dot{v}_C + 4v_C = 0, \quad v_C(0^+) = v_C(0^-) = 0 \text{ and } \dot{v}_C(0^+) = -60$$

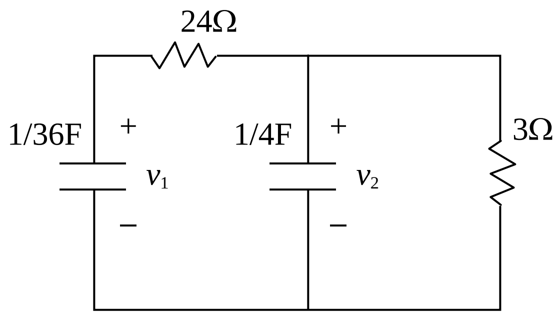
which results in eigenfunction $s^2 + 5s + 4 = 0 \Rightarrow s = -1, -4$, and then

$$v_C(t) = Ae^{-t} + Be^{-4t}, \quad v_C(0^+) = A + B = 0$$

$$\dot{v}_C(t) = -Ae^{-t} - 4Be^{-4t}, \quad \dot{v}_C(0^+) = -A - 4B = -60 \;.$$

Hence, $A = -20$ and $B = 20$, i.e., $v_C(t) = -20e^{-t} + 20e^{-4t}$, $t>0$.

The current $i(t) = \frac{v_C(t)}{4} = -5e^{-t} + 5e^{-4t}$, $t>0$.



4. If $v_1(0^-)=1$, $v_2(0^-)=1$, determine $v_1(t)$ for $t>0$.

Sol:

The component equations are

$$\frac{1}{36}\dot{v}_1 = i_{C1} = \frac{v_2 - v_1}{24} \Rightarrow \dot{v}_1(0^+) = 3\cdot\frac{v_2(0^+) - v_1(0^+)}{2} = 0$$

$$\frac{1}{4}\dot{v}_2 = i_{C2} = -i_{C1} - i = \frac{v_1 - v_2}{24} - \frac{v_2}{3} = \frac{v_1}{24} - \frac{3v_2}{8} \Rightarrow \dot{v}_2 = \frac{v_1}{6} - \frac{3v_2}{2}$$

The system equation is derived as

$$\dot{v}_1 = \frac{3}{2}(v_2 - v_1)$$

$$\Rightarrow \ddot{v}_1 = \frac{3}{2}(\dot{v}_2 - \dot{v}_1) = \frac{3}{2}\left(\left(\frac{v_1}{6} - \frac{3v_2}{2}\right) - \dot{v}_1\right) = \left(\frac{v_1}{4} - \frac{9v_2}{4}\right) - \frac{3}{2}\dot{v}_1$$

$$\Rightarrow \ddot{v}_1 + \frac{3}{2}\dot{v}_1 - \frac{v_1}{4} = -\frac{9}{4}v_2 = -\frac{9}{4}\left(\frac{2}{3}\dot{v}_1 + v_1\right) = -\frac{3}{2}\dot{v}_1 - \frac{9}{4}v_1$$

$$\Rightarrow \ddot{v}_1 + 3\dot{v}_1 + 2v_1 = 0$$

We have

$$\ddot{v}_1 + 3\dot{v}_1 + 2v_1 = 0, \quad v_1(0^+) = v_1(0^-) = 1 \text{ and } \dot{v}_1(0^+) = 0.$$

which results in eigenfunction $s^2 + 3s + 2 = 0 \Rightarrow s = -1, -2$, and then

$$v_1(t) = Ae^{-t} + Be^{-2t}, \quad v_1(0^+) = A + B = 1$$

$$\dot{v}_1(t) = -Ae^{-t} - 2Be^{-2t}, \quad \dot{v}_1(0^+) = -A - 2B = 0 \ .$$

Hence, $A = 2$ and $B = -1$. The total solution is $v_1(t) = 2e^{-t} - e^{-2t}$, $t>0$.