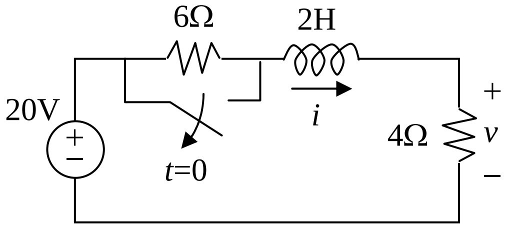
1. The switch has been closed for a long time and is opened at t=0. Solve for $v(t)$ for t>0.



Solution:

The component equation is $v_L = 2\frac{di}{dt}$.

The system equation is $20 = Ri + v_L = Ri + 2\frac{di}{dt} \Rightarrow \frac{di}{dt} + \frac{R}{2}i = 10$.

For t<0, we have R=4Ω and then $\frac{di}{dt} + 2i = 10$. Since the switch has been closed for a long time, it implies $i(0^-)$ is a constant and then $2i(0^-) = 10 \Rightarrow i(0^-) = 5$ A.

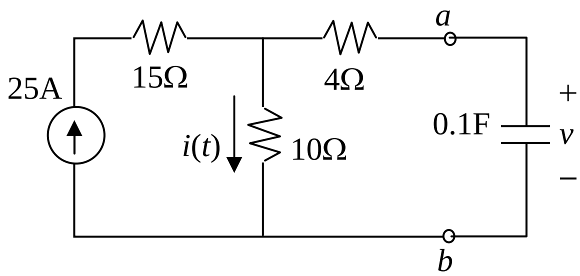
For t>0, we have R=4+6=10Ω and then $\frac{di}{dt} + 5i = 10$.

The current is solved as $i(t) = Ae^{-5t} + 2$.

From the initial condition $i(0^+)=i(0^-)=5$, we obtain A=3, i.e., $i(t) = 3e^{-5t} + 2$, for t>0.

This results in $v(t) = 4i(t) = 12e^{-5t} + 8$, for t>0.

2. Find the Thevenin equivalent circuit for the circuit to the left of terminals a and b. If $v(0^-)$=2 V, what is $i(t)$ for t>0?

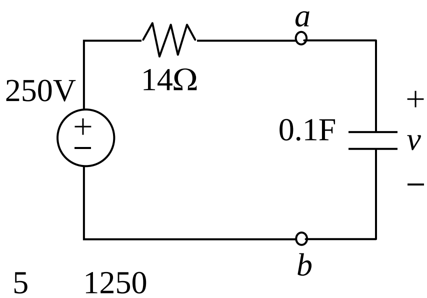


Solution:

The Thevenin equivalent resistance is $R_{Th} = 4 + 10 = 14\,\Omega$.

The open-circuit voltage is $v_{OC} = 10 \cdot 25 = 250$ V.

Hence, the Thevenin circuit is shown on the right.



The component equation is $i_C = 0.1\frac{dv}{dt}$.

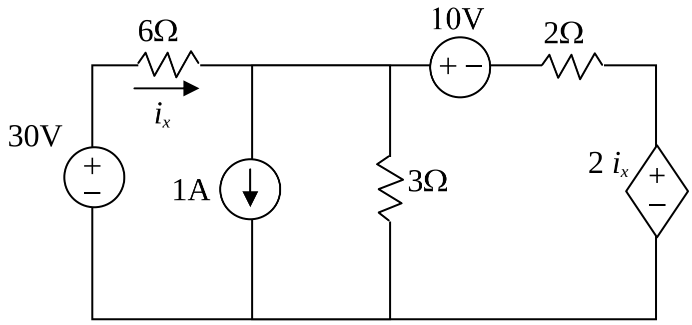
The system equation is $250 = 14i_C + v = 1.4\frac{dv}{dt} + v \Rightarrow \frac{dv}{dt} + \frac{5}{7}v = \frac{1250}{7}$.

For t>0, we have $v(t) = Ae^{-\frac{5}{7}t} + 250$.

From the initial condition $v(0^-)$=2, we obtain A=–248, i.e., $v(t) = -248e^{-\frac{5}{7}t} + 250$, for t>0.

This results in $i(t) = 25 - i_C(t) = 25 - 0.1 \cdot \left(\frac{1240}{7}e^{-\frac{5}{7}t}\right) = 25 - \frac{124}{7}e^{-\frac{5}{7}t}$, for t>0.

3. Use superposition to determine current i_x in the circuit on the right.

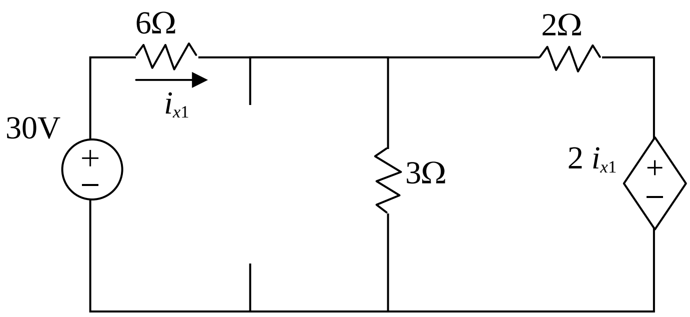


Solution:

(1) i_{x1}

$$-i_{x1}+\frac{30-6i_{x1}}{3}+\frac{30-6i_{x1}-2i_{x1}}{2}=0$$

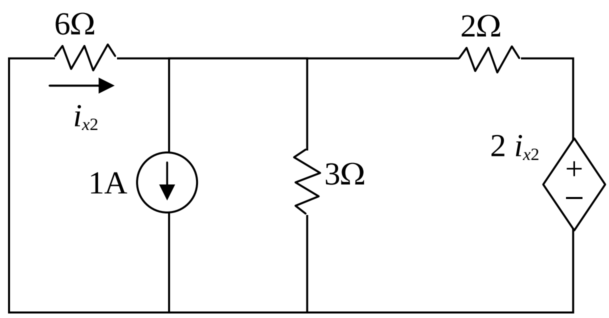
$$\Rightarrow i_{x1}=\frac{25}{7}$$



(2) i_{x2}

$$-i_{x2}+1+\frac{-6i_{x2}}{3}+\frac{-6i_{x2}-2i_{x2}}{2}=0$$

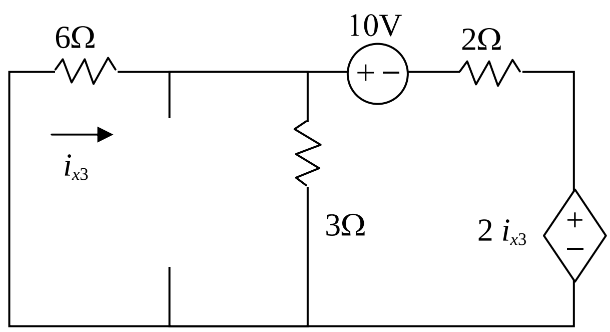
$$\Rightarrow i_{x2}=\frac{1}{7}$$



(3) i_{x3}

$$-i_{x3}+\frac{-6i_{x3}}{3}+\frac{-6i_{x3}-10-2i_{x3}}{2}=0$$

$$\Rightarrow i_{x3}=\frac{-5}{7}$$

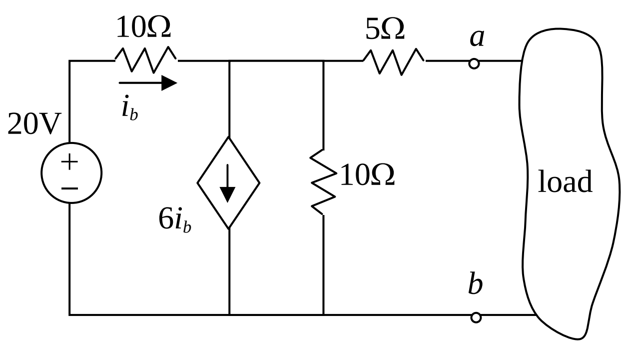


Hence,

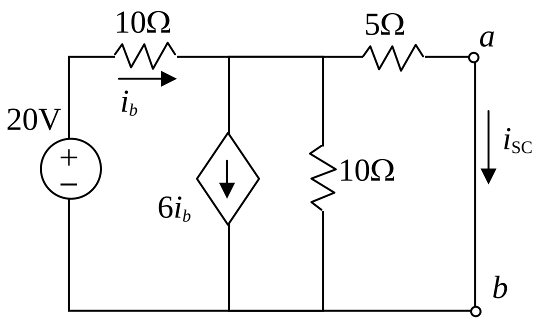
$$i_x=i_{x1}+i_{x2}+i_{x3}=\frac{25}{7}+\frac{1}{7}-\frac{5}{7}=3\text{ A}$$

4. (a) Determine the Norton equivalent circuit to the left of terminals a and b.

(b) If the load can absorb maximum power from the source, what is the equivalent resistance to the right of terminals a and b? What is the maximum power?



<u>Solution:</u>



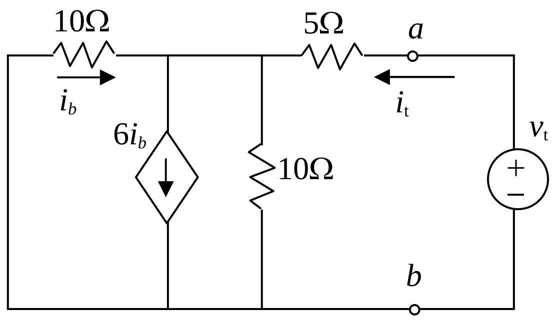
(a) Norton equivalent circuit is derived as below:

<i> i_{SC}:

$$i_b = \frac{20-5i_{SC}}{10},$$

$$-i_b + 6i_b + \frac{5i_{SC}}{10} + i_{SC} = 0 \Rightarrow i_b = -\frac{3}{10}i_{SC} = \frac{20-5i_{SC}}{10} \Rightarrow i_{SC} = 10\text{ A}$$

<ii> R_N (idle source)



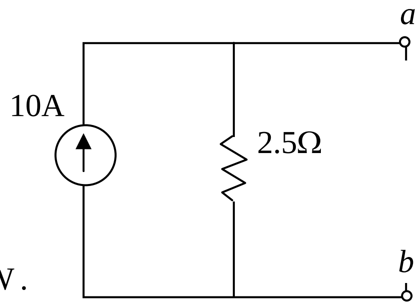
$$i_b = -\frac{v_t - 5i_t}{10}$$

$$-i_b + 6i_b + \frac{-10i_b}{10} - i_t = 0$$

$$\Rightarrow i_t = 4i_b = -\frac{2v_t - 10i_t}{5} \Rightarrow i_t = 0.4v_t$$

Hence, $R_N = \frac{v_t}{i_t} = 2.5\ \Omega$.

The Norton equivalent circuit is shown on the right.



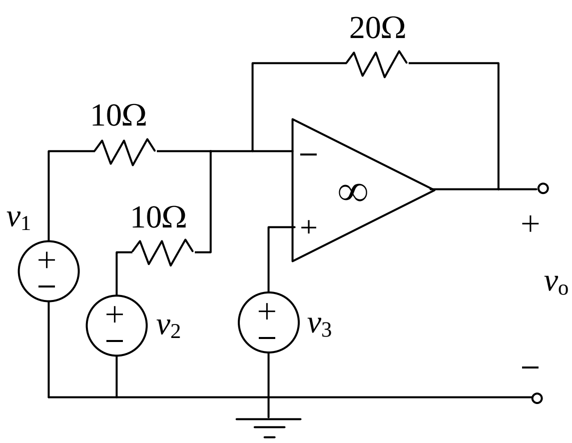
(b) The equivalent resistance of the load that absorbs maximum power is $R_L = R_N = 2.5\ \Omega$.

The maximum power is $R_L\left(10 \times \frac{2.5}{2.5+R_L}\right)^2 = 62.5\text{ W}$.

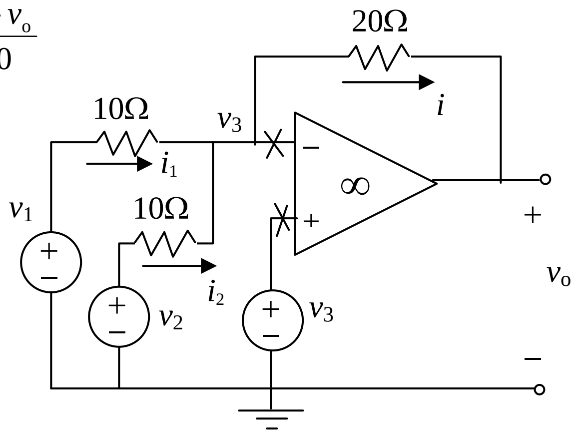
5. (a) Find the output voltages v_o in terms of v_1, v_2 and v_3.

(b) Let v_1=2*cos*(ωt), v_2=*sin*(2ωt) and v_3=*sin*(ωt–0.25π) in volt.

What is the effective value of v_0?



Solution:



(a) $i_1 = \dfrac{v_1 - v_3}{10}, \quad i_2 = \dfrac{v_2 - v_3}{10}, \quad i = \dfrac{v_3 - v_o}{20}$

Since $i = i_1 + i_2$, we have

$$\frac{v_3 - v_o}{20} = \frac{v_1 - v_3}{10} + \frac{v_2 - v_3}{10}$$

$$\Rightarrow v_o = -2v_1 - 2v_2 + 5v_3$$

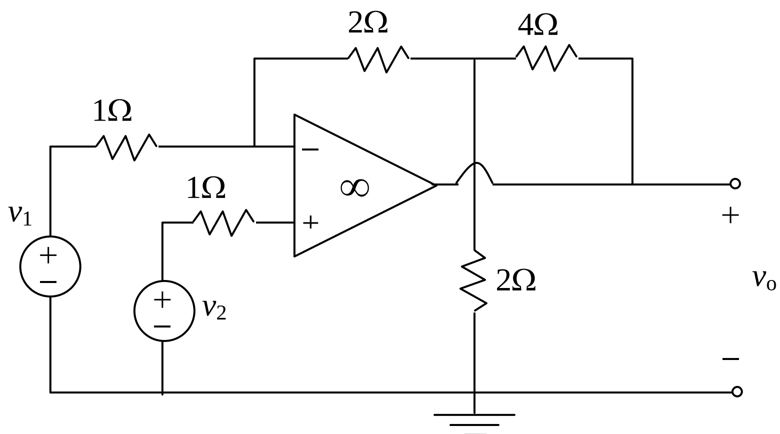
(b) The output voltage is

$$\begin{aligned} v_o &= -2v_1 - 2v_2 + 5v_3 \\ &= -4cos(\omega t) - 2\,sin(2\omega t) + 5\,sin(\omega t - 0.25\pi) \\ &= -4cos(\omega t) - 2\,sin(2\omega t) + \frac{5}{\sqrt{2}}[sin(\omega t) - cos(\omega t)] \\ &= -\left(4 + \frac{5\sqrt{2}}{2}\right)cos(\omega t) + \frac{5\sqrt{2}}{2}sin(\omega t) - 2\,sin(2\omega t) \end{aligned}$$

The effective value of v_0 is

$$V_{eff} = \sqrt{\frac{\left(4 + \frac{5\sqrt{2}}{2}\right)^2 + \left(\frac{5\sqrt{2}}{2}\right)^2 + 2^2}{2}} = \sqrt{\frac{45 + 20\sqrt{2}}{2}} = 6.0533$$

6. Find the output voltages v_o if v_1=2 V and v_2=1 V.

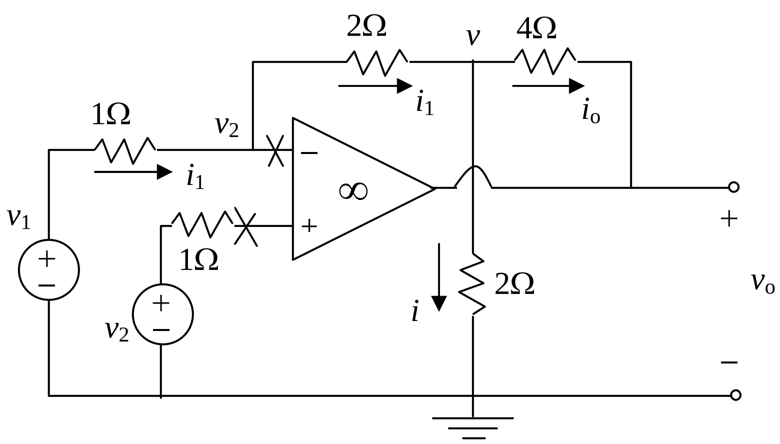


Solution:

$$i_1 = \frac{v_2 - v}{2} = \frac{v_1 - v_2}{1} = 1\,\text{A} \Rightarrow v = -1\,\text{V}$$

$$i = \frac{v}{2} = -0.5\,\text{A}, \quad i_o = \frac{v - v_o}{4} = i_1 - i = 1.5\,\text{A}$$

Hence, $v_o = -7\,\text{V}$



7. Consider the following first order differential equation:

$$\dot{y}(t)+2y(t)=u(t), \qquad y(0)=1.$$

(a) If $u(t)=t+2e^{-3t}$, determine $y(t)$ for t>0.

(b) If $u(t)=4sin(2t)$, determine $y(t)$ for t→∞.

Solution:

(a) The total solution is $y(t)=Ae^{-2t}+y_p(t)$, where

$$\dot{y}_p(t)+2y_p(t)=t+2e^{-3t}$$

Let $y_p(t)=\alpha t+\beta+\gamma e^{-3t}$, then $\dot{y}_p(t)=\alpha-3\gamma e^{-3t}$ and

$$\alpha-3\gamma e^{-3t}+2\alpha t+2\beta+2\gamma e^{-3t}=t+2e^{-3t}.$$

It results in $\alpha=\frac{1}{2}$, $\beta=-\frac{1}{4}$, $\gamma=-2$, i.e., $y_p(t)=\frac{1}{2}t-\frac{1}{4}-2e^{-3t}$.

Since $y(0)=1$, we have $y(0)=A+y_p(0)\Rightarrow 1=A-\frac{9}{4}\Rightarrow A=\frac{13}{4}$.

Hence, $y(t)=\frac{13}{4}e^{-2t}+\frac{1}{2}t-\frac{1}{4}-2e^{-3t}$.

(b) For t→∞, $y(t)=4\left|\frac{1}{j2+2}\right|sin\left(2t+\angle\frac{1}{j2+2}\right)=\sqrt{2}\,sin\left(2t-\frac{\pi}{4}\right)=sin(2t)-cos(2t)$