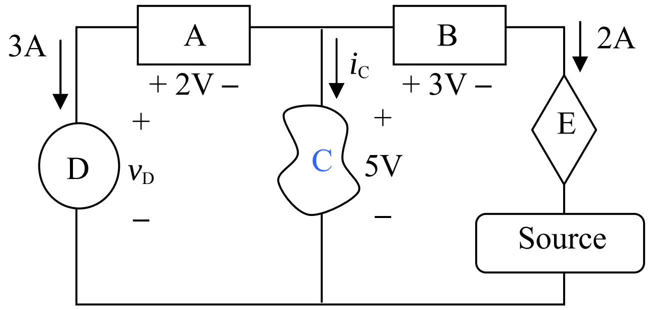
國立交通大學 95 暑修課程 電路一 Exam-1 (To Chap4) Solution

1. There are four fundamental *SI* units, m, kg, s, and A. Based on these fundamental units, some derived units are often used in electrical circuit analysis, such as C(coulomb), Ω(ohm), H(henry), and F(farad). Please write the SI units of C, Ω, H, and F.

Solution:

$$\mathrm{C} := \mathrm{A}\cdot\mathrm{s},\quad \Omega := \frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}^3\cdot\mathrm{A}^2},\quad \mathrm{H} := \frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}^2\cdot\mathrm{A}^2},\quad \mathrm{F} := \frac{\mathrm{s}^4\cdot\mathrm{A}^2}{\mathrm{kg}\cdot\mathrm{m}^2}$$

2. What is the absorbed power p_A of device A?
What is the absorbed power p_B of device B?.
What is the current i_C through device C?
What is the voltage drop v_D along device D?
If the power provided by the source is 10W, what is the absorbed power p_E of device E?



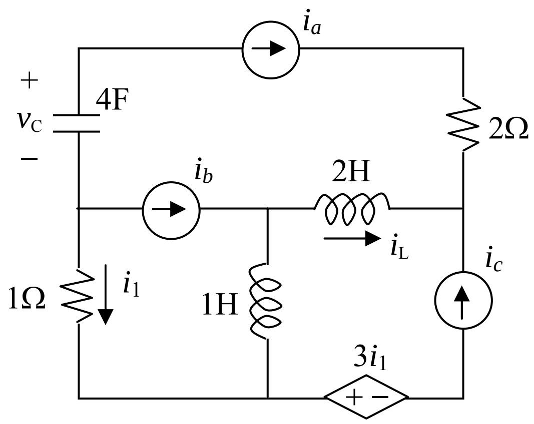
Solution:

$$p_A = (-3)\cdot 2 = -6\mathrm{W},\quad p_B = 2\cdot 3 = 6\mathrm{W},\quad i_C = (-3)+(-2) = -5\mathrm{A},\quad v_D = 2+5 = 7\mathrm{V}.$$

Since $p_C = (-5)\cdot 5 = -25\mathrm{W}$ and $p_D = 3\cdot 7 = 21\mathrm{W}$,

we have $p_E = 10-(p_A+p_B+p_C+p_D) = 10-(-6+6-25+21) = 14\mathrm{W}$

3. Determine v_C, i_L, and the power absorbed by the dependent source. The independent sources in amperes for $t\geq 0$ are $i_a=e^{-2t}$, $i_b=-3e^{-4t}$, and $i_c=5e^{-t}$. The initial capacitance voltage is $v_C(0)=1$V.



Solution:

$$v_C = v_C(0) + \frac{1}{C}\int_0^t i_C(\tau)dt = 1 + \frac{1}{4}\int_0^t(-i_a(\tau))dt$$

$$= 1 + \frac{1}{4}\int_0^t\left(-e^{-2\tau}\right)dt = 1 + \frac{1}{8}e^{-2\tau}\Big|_0^t = 1 + \frac{1}{8}e^{-2t} - \frac{1}{8} = \frac{7}{8} + \frac{1}{8}e^{-2t}$$

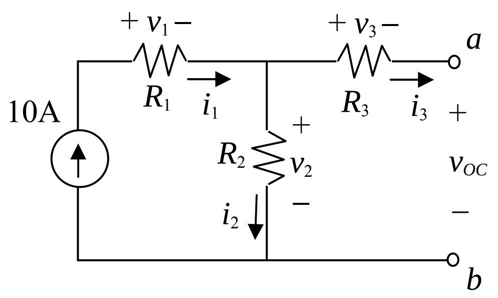
$$i_L = L\frac{di_L}{dt} = 2\cdot\frac{d(-i_a - i_c)}{dt} = 2\cdot\frac{d\left(-e^{-2t}-5e^{-t}\right)}{dt} = 4e^{-2t} + 10e^{-t}$$

The voltage drop across the dependent source is $v = 3i_1 = -3(i_a+i_b) = -3e^{-2t} + 9e^{-4t}$.

The current through it is $i = i_c = 5e^{-t}$.

So, the power absorbed is $vi = \left(-3e^{-2t}+9e^{-4t}\right)\cdot\left(5e^{-t}\right) = -15e^{-3t} + 45e^{-5t}$ W

4. For the circuit on the right, R_1=15Ω, R_2=10Ω, and R_3=10Ω.
   (a) Determine current i_1 and the open-circuit voltage v_{OC}. Also calculate the power absorbed by R_2 and R_3.
   (b) Short-circuit terminals a and b, then calculate current i_1, voltage v_2, and the short-circuit current i_{ab}.

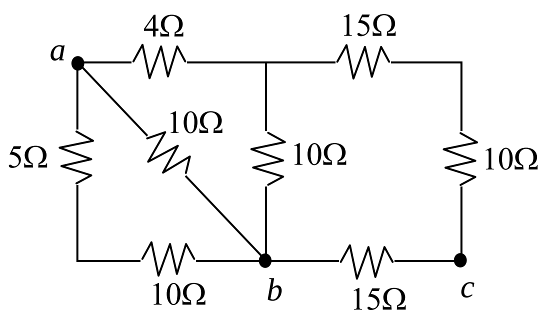


Solution:

(a) For the open circuit, i_3=0 and i_1= i_2.
Hence, $i_1 = i_2 = 10\text{A}$, $v_{OC} = v_2 = R_2 i_2 = 10 \cdot 10 = 100\text{V}$.
The power absorbed by R_2 is $v_2 i_2 = 100 \cdot 10 = 1000\text{W}$
The power absorbed by R_3 is $v_3 i_3 = 0\text{W}$.

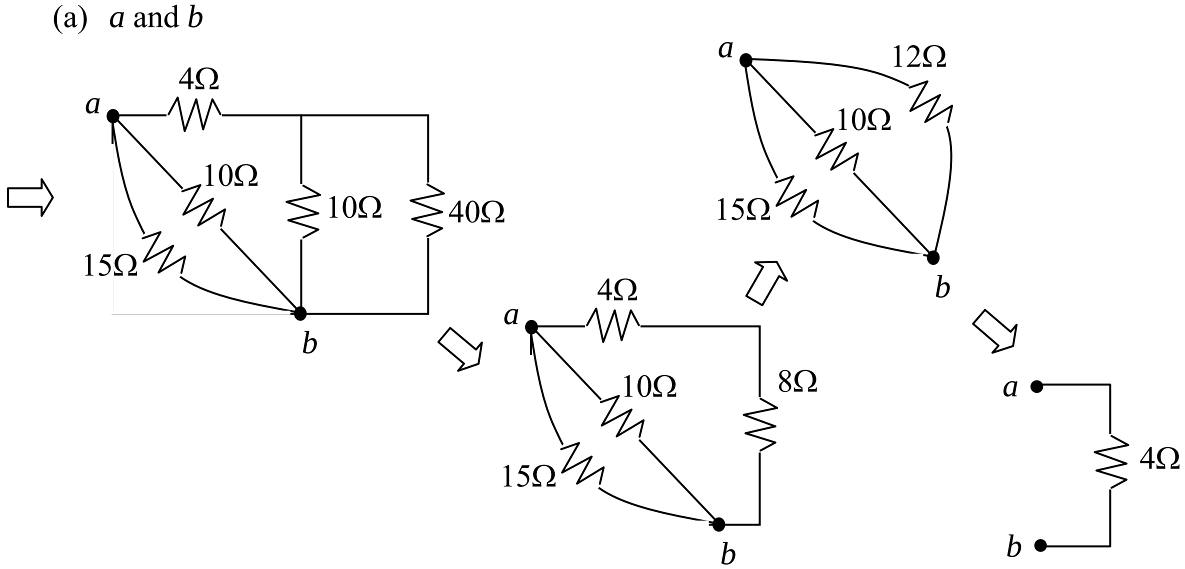
(b) For the short circuit, current i_1 does not change, that is, i_1=10A and i_2+i_3=10.
Because R_2=R_3 , we have i_2= i_3 and thus, i_2= i_3=5A.
The voltage drop $v_2 = R_2 i_2 = 10 \cdot 5 = 50\text{V}$.
The short-circuit current $i_{ab} = i_3 = 5\text{A}$.

5. Find the equivalent resistance when connections to the network shown on the right are made to the terminals specified.
   (a) a and b (b) b and c.

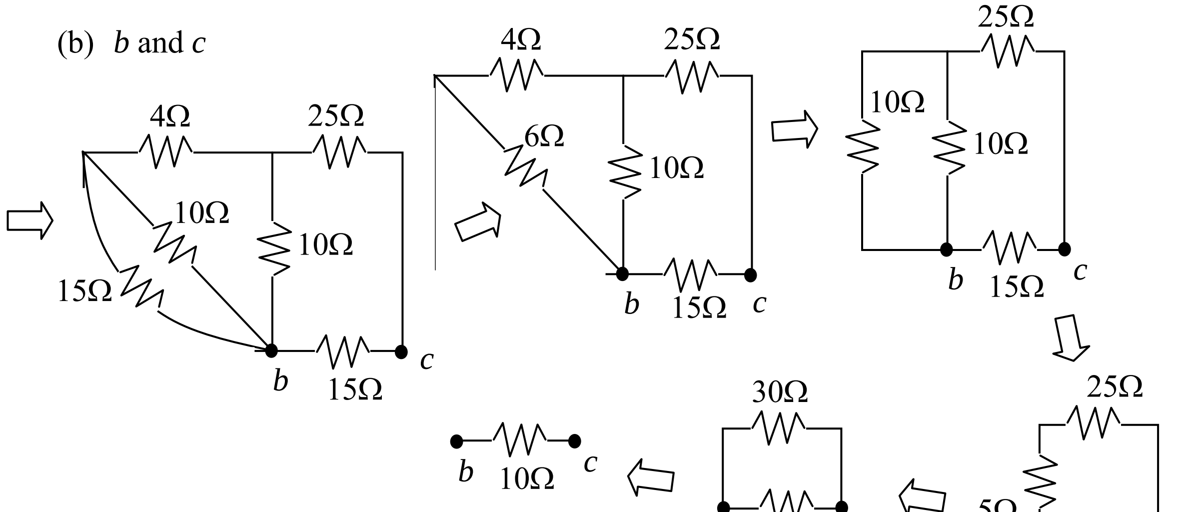


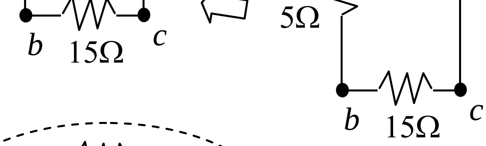
Solution:

(a) a and b

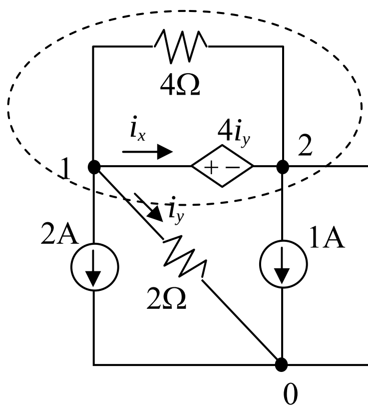


(b) b and c





6. Write a set of node-voltage equations in matrix form for the circuit on the right. Determine current i_x.



Solution:

Group v_1 and v_2 as the supernode.

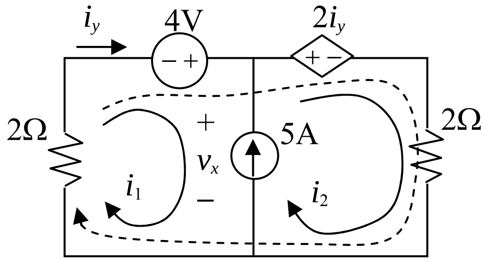
KVL: $v_1 - v_2 = 4i_y = 2v_1 \Rightarrow v_1 + v_2 = 0$

KCL: $2 + \frac{v_1}{2} + 1 + \frac{v_2}{4} = 0 \Rightarrow 2v_1 + v_2 = -12$

In matrix form, we obtain $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -12 \end{bmatrix}$ and thus, $v_1 = -12\text{V}$ and $v_2 = 12\text{V}$.

Hence, $i_x = -2 - \frac{v_1}{2} - \frac{v_1 - v_2}{4} = -2 + 6 + 6 = 10\text{A}$.

7. Write a set of mesh-current equations in matrix form for the circuit on the right. Determine voltage v_x.



Solution:

Group i_1 and i_2 as the supermesh.

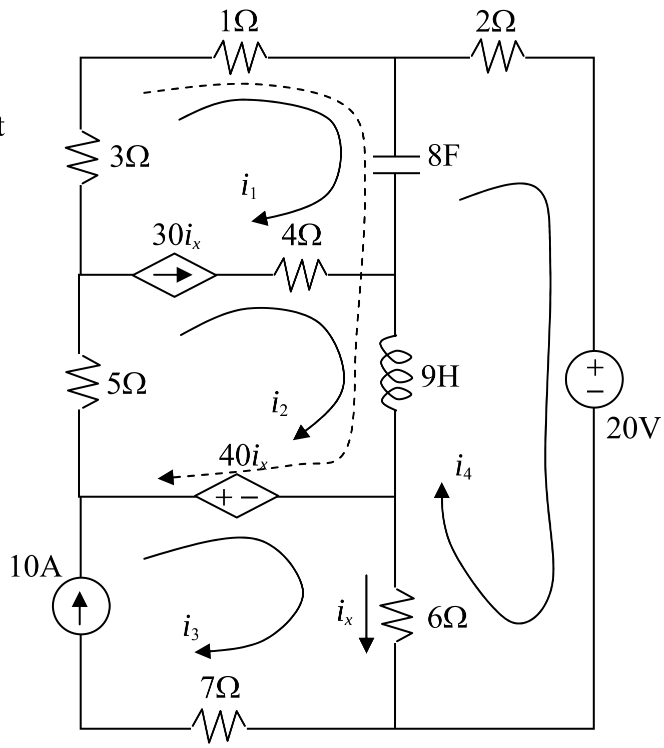
KCL: $i_2 - i_1 = 5 \Rightarrow i_1 - i_2 = -5$

KVL: $-4 + 2i_y + 2i_2 + 2i_1 = 0 \Rightarrow -4 + 2i_1 + 2i_2 + 2i_1 = 0 \Rightarrow 2i_1 + i_2 = 2$

In matrix form, we obtain $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$. The solution is $i_1 = -1\text{A}$ and $i_2 = 4\text{A}$.

Hence, $v_x = 4 - 2i_1 = 4 + 2 = 6\text{V}$.

8. For the circuit on the right, please define suitable mesh currents and then write a set of mesh-current equations in matrix form.



Solution:

The mesh currents are shown in the figure.

(1) KCL: $i_3 = 10$

(2) Group i_1 and i_2 as the supermesh

KCL: $i_2 - i_1 = 30 i_x = 30(i_3 - i_4)$

$$\Rightarrow i_1 - i_2 + 30 i_3 - 30 i_4 = 0$$

KVL: $-40 i_x + 5 i_2 + 3 i_1 + i_1 + \frac{1}{8}\int_{-\infty}^{t}(i_1 - i_4)\,d\tau + 9\frac{d(i_2 - i_4)}{dt} = 0$

$$\Rightarrow 4 i_1 + \frac{1}{8}\int_{-\infty}^{t} i_1\,d\tau + 5 i_2 + 9\frac{di_2}{dt} - 40 i_3 + 40 i_4 - \frac{1}{8}\int_{-\infty}^{t} i_4\,d\tau - 9\frac{di_4}{dt} = 0$$

(3) KVL: $20 + 6(i_4 - i_3) + 9\frac{d(i_4 - i_2)}{dt} + \frac{1}{8}\int_{-\infty}^{t}(i_4 - i_1)\,d\tau + 2 i_4 = 0$

$$\Rightarrow -\frac{1}{8}\int_{-\infty}^{t} i_1\,d\tau - 9\frac{di_2}{dt} - 6 i_3 + 8 i_4 + 9\frac{di_4}{dt} + \frac{1}{8}\int_{-\infty}^{t} i_4\,d\tau = -20$$

In matrix form, we have

$$\begin{bmatrix} 1 & -1 & 30 & -30 \\ 4 + \frac{1}{8}\int_{-\infty}^{t} d\tau & 5 + 9\frac{d}{dt} & -40 & 40 - \frac{1}{8}\int_{-\infty}^{t} d\tau - 9\frac{d}{dt} \\ 0 & 0 & 1 & 0 \\ -\frac{1}{8}\int_{-\infty}^{t} d\tau & -9\frac{d}{dt} & -6 & 8 + 9\frac{d}{dt} + \frac{1}{8}\int_{-\infty}^{t} d\tau \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \\ -20 \end{bmatrix}$$