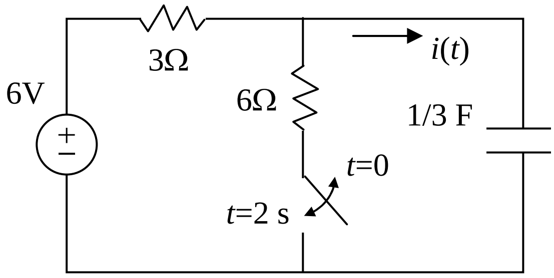
1. The switch has been closed for a long time. If the switch is opened at t=0 and is closed again at t=2 sec, determine $i(t)$ for t>0.



Sol:

Let the voltage across the capacitor is $v_C(t)$. Since the switch has been closed for a long time, we have $\frac{1}{3}\frac{dv_C}{dt}+\frac{v_C-6}{3}+\frac{v_C}{6}=0$, where $\frac{dv_C}{dt}(0^-)=0$ and then $v_C(0^-)=4$V.

For 2>t>0, the switch is opened and then $\frac{1}{3}\frac{dv_C}{dt}+\frac{v_C-6}{3}=0 \Rightarrow \frac{dv_C}{dt}+v_C=6$.

This leads to $v_C(t)=Ae^{-t}+6$ and $v_C(0^+)=v_C(0^-)=4 \Rightarrow A+6=4 \Rightarrow A=-2$.

Hence, $v_C(t)=-2e^{-t}+6$ and $i(t)=\frac{1}{3}\dot{v}_C(t)=\frac{2}{3}e^{-t}$ for 2>t>0.

For t>2, the switch is closed again and then $\frac{1}{3}\frac{dv_C}{dt}+\frac{v_C-6}{3}+\frac{v_C}{6}=0$, where $v_C(2^-)=-2e^{-2}+6$.

We have $\frac{dv_C}{dt}+\frac{3v_C}{2}=6$ and then $v_C(t)=Be^{-\frac{3}{2}t}+4$.

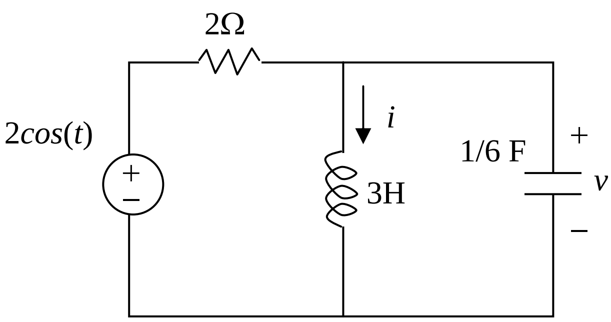
Hence, $v_C(2^+)=v_C(2^-)=-2e^{-2}+6 \Rightarrow Be^{-3}+4=-2e^{-2}+6 \Rightarrow B=-2e+2e^3$.

This results in $v_C(t)=(-2e+2e^3)e^{-\frac{3}{2}t}+4=34.73e^{-\frac{3}{2}t}+4$ and $i(t)=\frac{1}{3}\dot{v}_C(t)=-17.37e^{-\frac{3}{2}t}$.

The total solution for t>0 is

$$i(t)=\begin{cases}\frac{2}{3}e^{-t}, & 2>t>0\\ -17.37e^{-\frac{3}{2}t}, & t>2\end{cases}$$

2. Determine $v(t)$ for $t\to\infty$.



Sol:

The component equations are $\frac{1}{6}\frac{dv}{dt}=i_C=\frac{2cos(t)-v}{2}-i$ and $3\frac{di}{dt}=v_L=v$, i.e.,

$$\dot{v}=-3v-6i+6cos(t) \text{ and } \frac{di}{dt}=\frac{1}{3}v.$$

Hence, the system equation is derived as

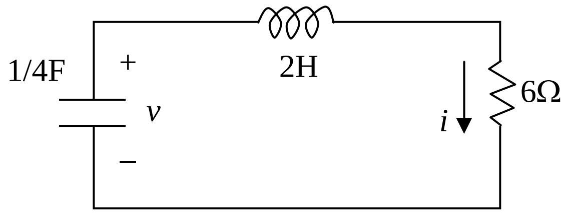
$$\ddot{v} = -3\dot{v} - 6\frac{di}{dt} - 6\,sin(t) = -3\dot{v} - 2v - 6\,sin(t) \Rightarrow \ddot{v} + 3\dot{v} + 2v = -6\,sin(t).$$

First, calculate $\left|\frac{1}{D^2+3D+2}\right|_{D=j} = \left|\frac{1}{1+j3}\right| = \frac{1}{\sqrt{10}}\angle\theta$, where $cos\,\theta = \frac{1}{\sqrt{10}}$ and $sin\,\theta = -\frac{3}{\sqrt{10}}$.

Hence,

$$v(t) = -6\cdot\left(\frac{1}{\sqrt{10}}\right)sin(t+\theta) = -\frac{6}{\sqrt{10}}\left(sin(t)cos\,\theta + cos(t)sin\,\theta\right) = -\frac{6}{10}sin(t) + \frac{18}{10}cos(t).$$

3. If $i(0^-)=1$, $v(0^-)=2$, determine $v(t)$ for $t>0$.



Sol:

The component equations are

$$\frac{1}{4}\dot{v} = -i \Rightarrow \dot{v}(0^+) = -4i(0^+) = -4i(0^-) = -4$$

$$2\frac{di}{dt} = v - 6i$$

The system equation is derived as $\ddot{v} = -4\frac{di}{dt} = -2v + 12i = -2v - 3\dot{v} \Rightarrow \ddot{v} + 3\dot{v} + 2v = 0$,

i.e., $\ddot{v} + 3\dot{v} + 2v = 0$, $v(0^+) = v(0^-) = 2$ and $\dot{v}(0^+) = -4$.

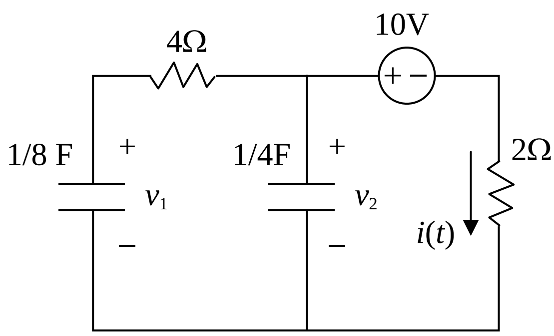
The eigenfunction is $s^2 + 3s + 2 = 0 \Rightarrow s = -1, -2$, and then

$$v(t) = Ae^{-t} + Be^{-2t}, \quad v(0^+) = A + B = 2$$

$$\dot{v}(t) = -Ae^{-t} - 2Be^{-2t}, \quad \dot{v}(0^+) = -A - 2B = -4.$$

Hence, $A = 0$ and $B = 2$, i.e., $v(t) = 2e^{-2t}$, $t>0$.

4. If $v_1(0^-)=0$, $v_2(0^-)=10$, determine $i(t)$ for $t>0$.



Sol:

The component equations are

$$\frac{1}{8}\dot{v}_1 = i_{C1} = \frac{v_2 - v_1}{4} \Rightarrow \dot{v}_1 = 2v_2 - 2v_1$$

$$\frac{1}{4}\dot{v}_2 = i_{C2} = -i_{C1} - i = \frac{v_1 - v_2}{4} - \frac{v_2 - 10}{2} = \frac{v_1}{4} - \frac{3v_2}{4} + 5 \Rightarrow \dot{v}_2 = v_1 - 3v_2 + 20$$

The system equation is derived as

$$\ddot{v}_2 = \dot{v}_1 - 3\dot{v}_2 = 2v_2 - 2v_1 - 3\dot{v}_2 = 2v_2 - 2(\dot{v}_2 + 3v_2 - 20) - 3\dot{v}_2 = -5\dot{v}_2 - 4v_2 + 40$$

We have

$$\ddot{v}_2 + 5\dot{v}_2 + 4v_2 = 40.$$

where $v_2(0^+) = v_2(0^-) = 10$ and $\dot{v}_2(0^+) = v_1(0^+) - 3v_2(0^+) + 20 = v_1(0^-) - 3v_2(0^-) + 20 = -10$

The particular solution is v_{2p}=10. The eigenfunction is $s^2 + 5s + 4 = 0 \Rightarrow s = -1,\ -4$, and then

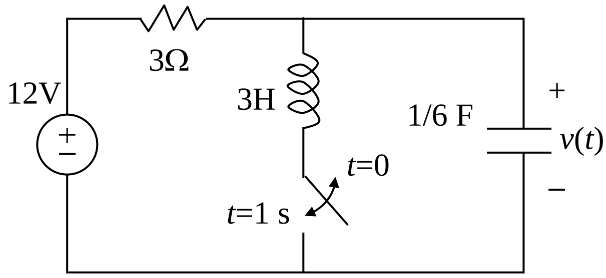
$$v_2(t) = Ae^{-t} + Be^{-4t} + 10, \qquad v_2(0^+) = A + B + 10 = 10$$

$$\dot{v}_2(t) = -Ae^{-t} - 4Be^{-4t}, \qquad \dot{v}_2(0^+) = -A - 4B = -10 \ .$$

Hence, $A = -\frac{10}{3}$ and $B = \frac{10}{3}$, which results in $v_2(t) = -\frac{10}{3}e^{-t} + \frac{10}{3}e^{-4t} + 10$, t>0.

The solution is $i(t) = \frac{v_2(t) - 10}{2} = -\frac{5}{3}e^{-t} + \frac{5}{3}e^{-4t}$, t>0.

5. The switch has been closed for a long time. If the switch is opened at t=0 and is closed again at t=1 sec, determine $v(t)$ for t>0.



Sol:

Let the current through the inductor is $i_L(t)$. Since the switch has been closed for a long time, we have constant $i_L(t)$ at $t=0^-$, i.e., $v(0^-) = L\frac{di_L(t)}{dt}\bigg|_{t=0^-} = 0$.

For 1>t>0, the switch is opened and then $\frac{1}{6}\frac{dv}{dt} + \frac{v-12}{3} = 0 \Rightarrow \dot{v} + 2v = 24$.

This leads to $v(t) = Ae^{-2t} + 12$ and $v(0^+) = v(0^-) = 0 \Rightarrow A + 12 = 0 \Rightarrow A = -12$.

Hence, $v(t) = -12e^{-2t} + 12$ for 1>t>0 and $v(1^-) = -12e^{-2} + 12 = 10.376 = v(1^+)$.

For t>1, with the switch closed again, we have a second-order circuit.

The component equations are $\frac{1}{6}\frac{dv}{dt} = i_C = \frac{12 - v}{3} - i_L$ and $3\frac{di_L}{dt} = v_L = v$, i.e.,

$$\dot{v} = -2v - 6i_L + 24 \quad \text{and} \quad \frac{di_L}{dt} = \frac{1}{3}v.$$

The initial conditions are $i_L(1^+) = 0$ and $\dot{v}(1^+) = -2v(1^+) - 6i_L(1^+) + 24 = -2(10.376) + 24 = 3.248$.

Hence, the system equation is derived as

$$\ddot{v} = -2\dot{v} - 6\frac{di_L}{dt} = -2\dot{v} - 2v \Rightarrow \ddot{v} + 2\dot{v} + 2v = 0 .$$

The eigenfunction is $s^2 + 2s + 2 = 0 \Rightarrow s = -1 \pm j$, and then

$$v(t) = e^{-t}\left(Acos(t) + Bsin(t)\right),$$
$$v(1^+) = e^{-1}\left(Acos(1) + Bsin(1)\right) = 10.376 \Rightarrow 0.540A + 0.841B = 28.205$$
$$\dot{v}(t) = -e^{-t}\left(Acos(t) + Bsin(t)\right) + e^{-t}\left(-Asin(t) + Bcos(t)\right),$$
$$\dot{v}(1^+) = e^{-1}\left(-A[cos(1) + sin(1)] + B[cos(1) - sin(1)]\right) = 3.248 \Rightarrow 1.382A + 0.301B = -8.829 .$$

Hence, $A = -15.933$ and $B = 43.779$, i.e., $v(t) = e^{-t}\left(-15.933cos(t) + 43.779sin(t)\right)$, $t>1$.

The total solution is $v(t) = -12e^{-2t} + 12$

$$v(t) = \begin{cases} -12e^{-2t} + 12, & 1 > t > 0 \\ e^{-t}\left(-15.933cos(t) + 43.779sin(t)\right), & t > 1 \end{cases}$$