NCTU Summer Course <Electrical Circuits> Quiz-01

________大學 ___________系 學號_____________ 姓名_________________

1. (25%) In addition to the fundamental SI units m, kg, sec and A, some derived units are also used in electrical engineering. For example, J(joule) is the unit of energy *E*, W(watt) is the unit of power *P*, H(henry) is the unit of inductance *L* and Ω(Ohm) is the unit of resistance *R*. It is known that the power dissipated in a resistor is $P = RI^2$ and the energy stored in an inductor is $E = LI^2/2$ where *I* is the current. Show the expression of J, W, H and Ω in terms of m, kg, sec and A.

Sol:

$\mathrm{J} \equiv \mathrm{kg{\cdot}m^2/s^2}$

$\mathrm{W} \equiv \mathrm{kg{\cdot}m^2/s^3}$

$\mathrm{H} \equiv \mathrm{kg{\cdot}m^2/s^2/A^2}$

$\Omega \equiv \mathrm{kg{\cdot}m^2/s^3/A^2}$

2. (25%) Let $f(t) = \int_0^2 (2x^2+3)dx$, $g(t) = \int_0^t (2y^2+3)dy$ and $h(t) = \int_0^t (2tz+3)dz$.

Calculate $\frac{df(t)}{dt}$, $\frac{dg(t)}{dt}$ and $\frac{dh(t)}{dt}$.

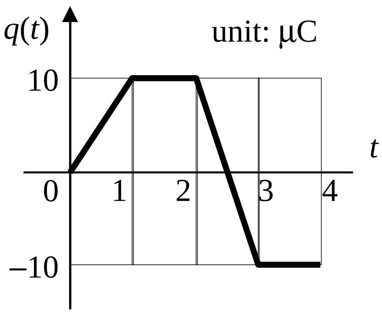
Sol:

$$f(t) = \int_0^2 (2x^2+3)dx = \text{constant} \;\Rightarrow\; \frac{df(t)}{dt} = 0$$

$$g(t) = \int_0^t (2y^2+3)dy \;\Rightarrow\; \frac{dg(t)}{dt} = 2t^2+3$$

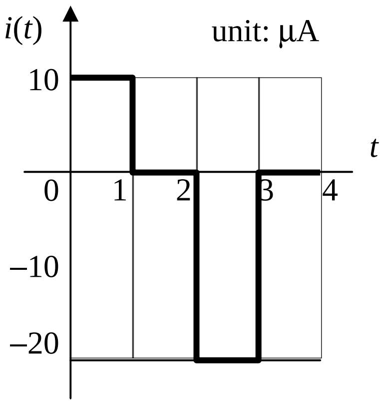
$$h(t) = \int_0^t (2tz+3)dz = t^3+3t \;\Rightarrow\; \frac{dh(t)}{dt} = 3t^2+3$$

3. (25%) Consider an electrical element whose voltage $v(t)=\alpha q(t)$ where α=2000V/C and $q(t)$ is the stored charge as shown in the figure on the right. Let the current through the element be $i(t)=\frac{dq(t)}{dt}$ in passive sign. What is the average power for $t \in [0,4]\text{sec}$?



<u>Sol:</u>

$$P=\frac{1}{4}\int_0^4 p(t)dt=\frac{1}{4}\int_0^4 v(t)i(t)dt$$
$$=500\int_0^4 q(t)i(t)dt$$
$$=5\times10^{-10}\left(\int_0^1(10t\cdot10)dt+\int_2^3(-20(t-2.5)\cdot(-20))dt\right)$$
$$=5\times10^{-8}(0.5+0)=2.5\times10^{-8}\,\text{W}$$



4. (25%) Assume a long conductor with cross area $a=9\times10^{-6}\,\text{m}^2$ is uniformly distributed with electrical particles whose density is $n=5\times10^{28}$ particles$/\text{m}^3$. Let the charge of each particle be $q=3.2\times10^{-19}\,\text{C}$. If a current $i(t)$=0.2A through the conductor is measured, what is the velocity of the electrical particles?

<u>Sol:</u>

$$I=\frac{\Delta Q}{\Delta t}=\frac{\Delta Q/L}{\Delta t/L}=\frac{\Delta Q\cdot v}{L}=\frac{aLnq\cdot v}{L}=anqv$$

Hence,

$$v=\frac{I}{anq}=\frac{0.2}{9\times10^{-6}\times5\times10^{28}\times3.2\times10^{-19}}=\frac{1}{9\times10^{4}\times8}=1.39\times10^{-6}\text{ m/s}$$

NCTU Summer Course <Electrical Circuits> Quiz-02

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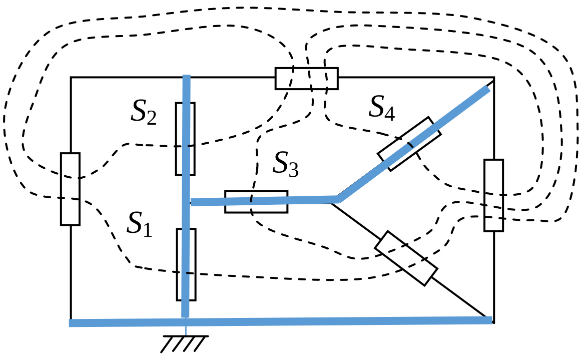
1. (25%) In the following circuit,

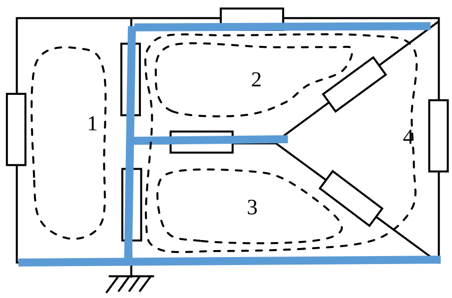
   (A) select a tree and show the cut surfaces S_i, i=1,2,...,

   (B) select a different tree and show the link loops, $_i$, i=1,2,...

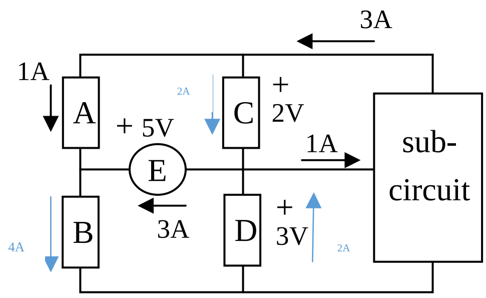
Sol:

(A) (B)





2. (25%) A circuit is depicted on the right, which contains five elements and one sub-circuit. Determine the power absorbed by each element. Is the sub-circuit a power provider or a power absorber?



Sol:

$V_A = V_E - V_C = -(5-2) = -3V$

$P_A = -3 \times 1 = -3W$

$V_B = -(-V_E + V_D) = 8V$

$P_B = 8 \times 4 = 32W$

$P_C = 2 \times 2 = 4W$

$P_D = -2 \times 3 = -6W$

$V_E = -3 \times 5 = -15W$

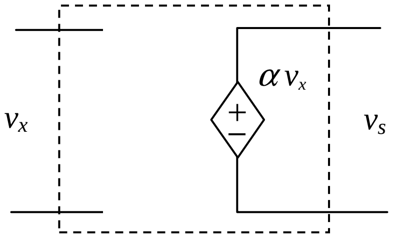
$P_A + P_B + P_C + P_D + P_E + P_{sub} = 0$

$P_{sub} = -12W$

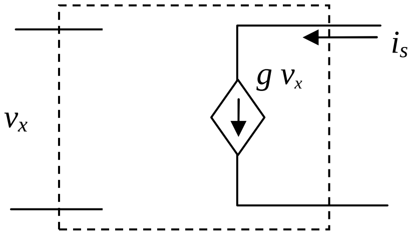
(power provider)

3. (25%) There are four dependent sources, voltage-controlled voltage source (VCVS), voltage-controlled current source (VCCS), current-controlled voltage source (CCVS), and current-controlled current source (CCCS). Please write their component symbols and control equations.

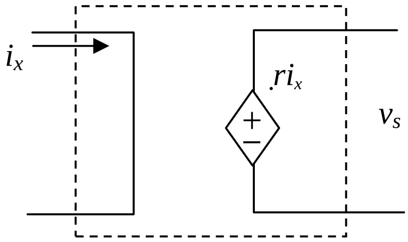
Sol:



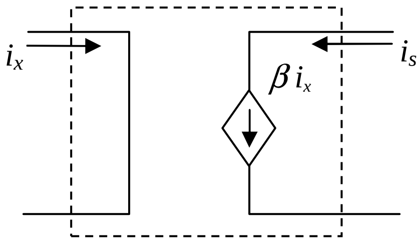
(a) VCVS



(b) VCCS

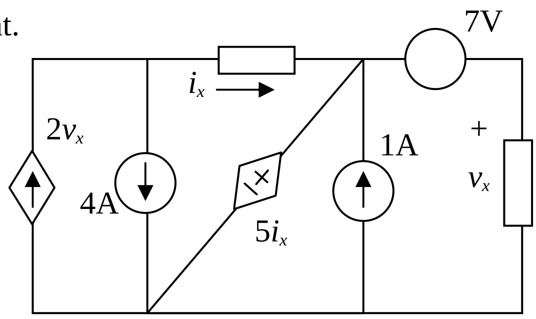


(c) CCVS



(d) CCCS

4. (25%) A circuit is shown on the right. Find the current i_x and voltage v_x.



Sol:

$2v_x = i_x + 4$

$5i_x + (-7V) - v_x = 0$

$v_x = 3V$

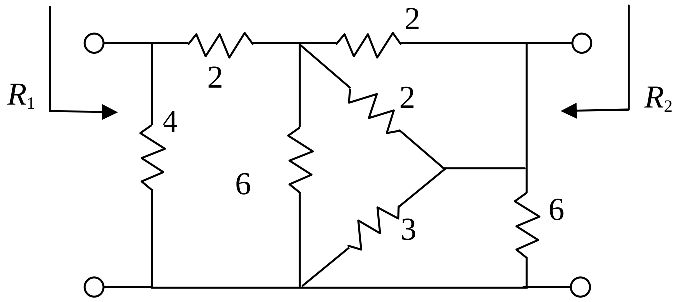
$i_x = 2A$

NCTU Summer Course <Electrical Circuits> Quiz-03

________大學 ___________系 學號_____________ 姓名_________________

1. (25%) In the following circuit, Find the equivalent resistances R_1 and R_2.

Sol:

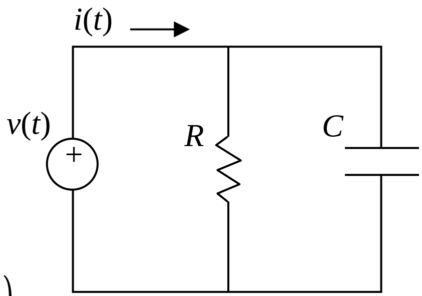


R_1: ((((6 // 3)+(2 // 2)) // 6)+2) // 4 = 2

R_2: ((4+2) // 6) // ((2 // 2)+3) // 6 = 4/3

2. (25%) A voltage source $v(t)=3\cos 3t + 2e^{-2t}$ V is applied to a capacitor with C=47μF parallel to a resistor R=2kΩ.What is the current $i(t)$ generated by the voltage source?

Sol:



$i(t) = i_C(t) + i_R(t)$

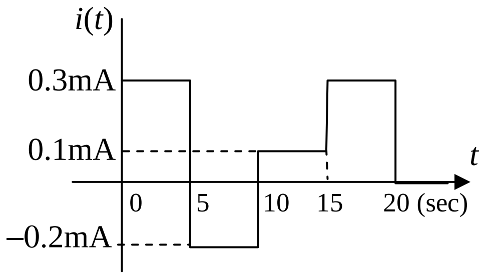
$i_R(t) = \dfrac{3\cos 3t + 2e^{-2t}}{2000} A$

$i_C(t) = C\dfrac{dv(t)}{dt} = 47\times10^{-6}\left(-9\sin 3t - 4e^{-2t}\right)$

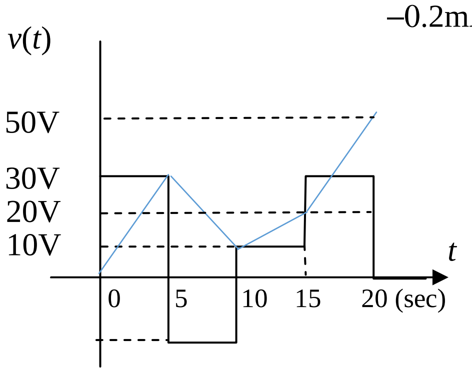
Hence,

$$i(t) = \frac{3\cos 3t + 2e^{-2t}}{2000} + 47\times10^{-6}\left(-9\sin 3t - 4e^{-2t}\right)$$
$$= 1.5\cos 3t - 0.423\sin 3t + 0.812e^{-2t} \text{ mA}$$

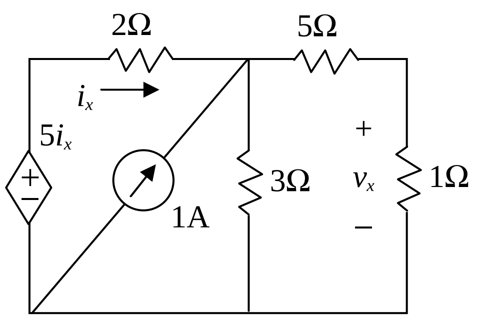
3. (25%) The current $i(t)$ through a capacitor with C=50μF is shown in the following figure. If the capacitor has no charge at the initial time, i.e., $q_c(0)$=0 C, draw the voltage $v(t)$ of the capacitor.



Sol:



4. (25%) A circuit is shown on the right. Find the current i_x and voltage v_x.



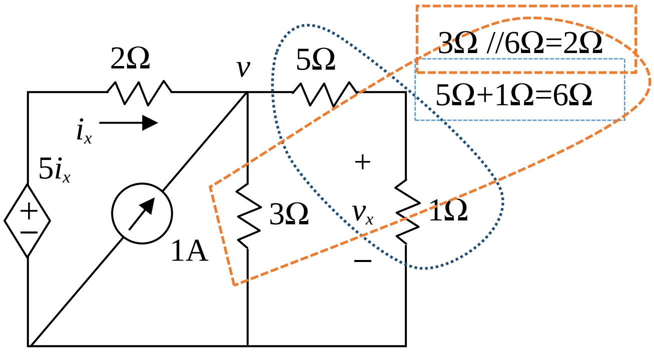
Sol:

$5i_x = 2\ i_x + (1 + i_x)2$

i_x=2 A

$v=5i_x - 2i_x = 10–4 = 6V$

$$v_x = \frac{1}{6} v = 1\,\text{V}$$

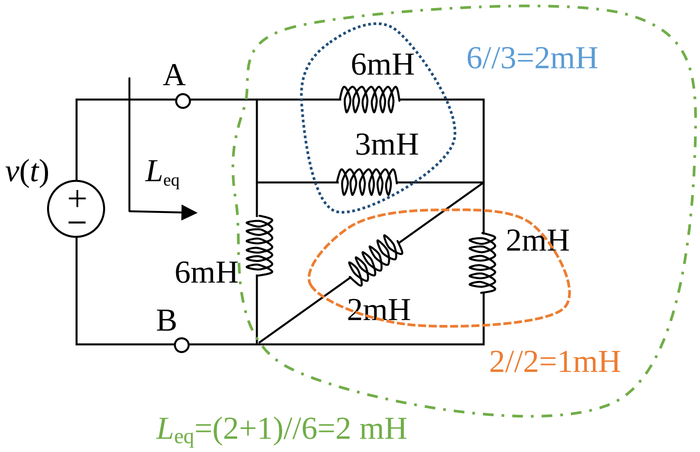


NCTU Summer Course <Electrical Circuits> Quiz-04

________大學 ___________系 學號_____________ 姓名_________________

1. (30%) The voltage source is $v(t)=0.6\,sin2t$ and there is no initial energy stored in the circuit. Find the equivalent inductance L_{eq} through A and B and the total energy $w(t)$ stored in all the inductors at time t.

Sol:



L_{eq}= 2mH

$v(t)=0.6\,sin2t$

$$i_L(t)=\frac{1}{L_{eq}}\int_0^t v(\tau)d\tau$$

$$=\frac{10^3}{2}\int_0^t 0.6\,sin\,2\tau\,d\tau$$

$$=-\frac{10^3}{2}0.3cos\,2\tau\Big|_0^t$$

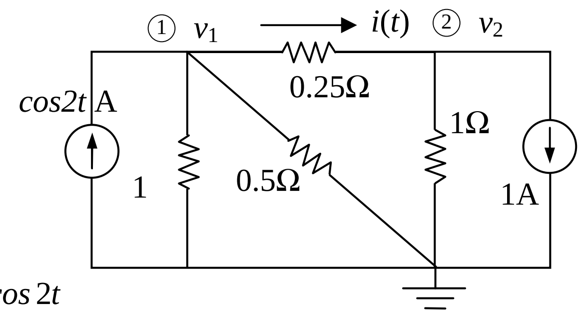
$$=-0.15(cos\,2t-1)\ \text{kA}$$

$$w(t)=\frac{1}{2}L_{eq}i_L^2(t)$$

$$=22.5(cos\,2t-1)^2$$

2. (30%) Based on nodal analysis, determine the current $i(t)$.

Sol:



KCL①: $4(v_1-v_2)+1\cdot v_1+2\cdot v_1=cos\,2t$

⇨ $7v_1-4v_2=cos2t$

KCL②: $4(v_2-v_1)+1\cdot v_2=-1$

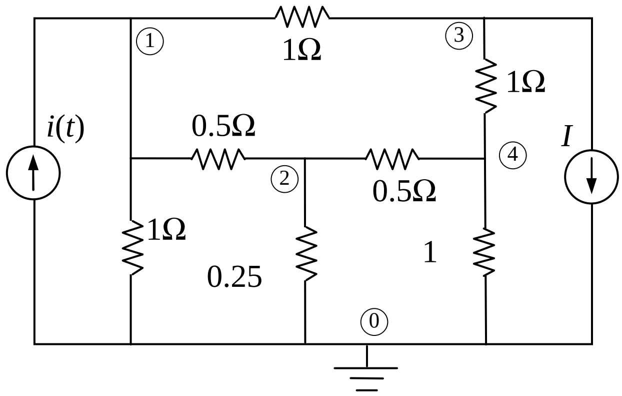
⇨ $-4v_1+5v_2=-1$

Hence, $v_1=\dfrac{-4+5cos\,2t}{19}$, $v_2=\dfrac{-7+4cos\,2t}{19}$

⇨ $i(t)=4(v_1-v_2)=\dfrac{12+4cos\,2t}{19}$ A

3. (40%) Based on nodal analysis, write the nodal voltage equation ***Fv***=***Gu***.

Sol:



$KCL①$: $1\cdot(v_1 - v_3) + 1\cdot v_1 + 2\cdot(v_1 - v_2) = i(t)$

⇨ $4v_1 - 2v_2 - v_3 = i(t)$

$KCL②$: $2\cdot(v_2 - v_1) + 4\cdot v_2 + 2\cdot(v_2 - v_4) = 0$

⇨ $-2v_1 + 8v_2 - 2v_4 = 0$

$KCL③$: $1\cdot(v_3 - v_1) + 1\cdot(v_3 - v_4) = -I$

⇨ $-v_1 + 2v_3 - v_4 = -I$

$KCL④$: $1\cdot(v_4 - v_3) + 1\cdot v_4 + 2\cdot(v_4 - v_2) = 0$

⇨ $-2v_2 - v_3 + 4v_4 = 0$

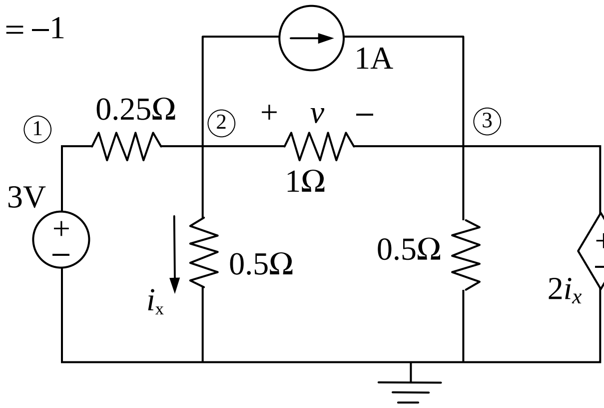
Hence, we have

$$\underbrace{\begin{bmatrix} 4 & -2 & -1 & 0 \\ -2 & 8 & 0 & -2 \\ -1 & 0 & 2 & -1 \\ 0 & -2 & -1 & 4 \end{bmatrix}}_{\boldsymbol{F}} \cdot \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}}_{\boldsymbol{v}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}}_{\boldsymbol{G}} \cdot \underbrace{\begin{bmatrix} i(t) \\ I \end{bmatrix}}_{\boldsymbol{u}}$$

NCTU Summer Course <Electrical Circuits> Quiz-05

________大學 ___________系 學號_____________ 姓名_________________

1. (50%) Based on the nodal analysis, solve the voltage v across the resistor 1Ω.



Sol:

KVL①: $v_1 = 3$

KCL②: $4(v_2 - v_1) + 2v_2 + 1\cdot(v_2 - v_3) = -1$

⇨ $-4v_1 + 7v_2 - v_3 = -1$

KCL③: $v_3 = 2(2v_2) = 4v_2$

We have

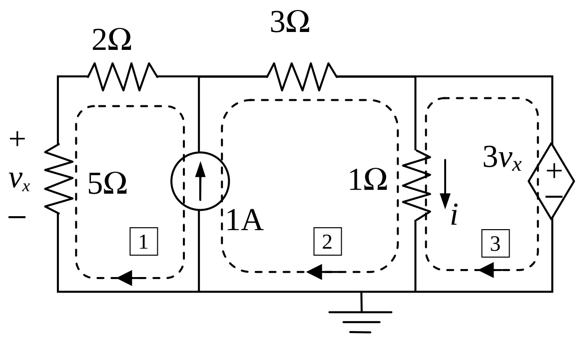
$$-12 + 7v_2 - 4v_2 = -1$$

⇨ $v_2 = \dfrac{11}{3}$

and then

⇨ $v = v_2 - v_3 = -3v_3 = -11\,\text{V}$

2. (50%) Based on the mesh analysis, solve the current i through the resistor 1Ω.



Sol:

KCL[1][2]: $i_2 - i_1 = 1$

KVL[1][2]: $5i_1 + 2i_1 + 3i_2 + 1\cdot(i_2 - i_3) = 0$

⇨ $7i_1 + 4i_2 - i_3 = 0$

KVL[3]: $3(-5i_1) + 1\cdot(i_3 - i_2) = 0$

⇨ $-15i_1 - i_2 + i_3 = 0$

We have

$$i_2 - i_1 = 1$$

$$-8i_1 + 3i_2 = 0$$

which leads to $i_1 = 0.6$, $i_2 = 1.6$, and $i_3 = 10.6$.

Therefore, $i = i_2 - i_3 = 9\,\text{A}$

NCTU Summer Course <Electrical Circuits> Quiz-06

________大學 ___________系 學號_____________ 姓名_________________

1. (50%) Draw the Norton equivalent circuit referring to the load resistance R_L.

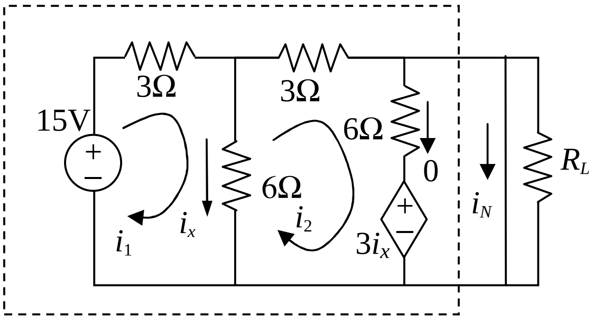
<u>Sol:</u>

(I) Short R_L to find short current i_N

Since the output is short,

We have $i_x = 0$.

Then, $i_N = 15/(3+3) = 2.5\text{ A}$



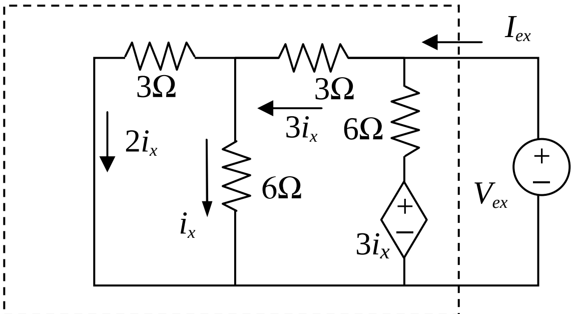
(II) Take away source and add an extra source

$$V_{ex} = 3\cdot 3i_x + 6i_x = 6(I_{ex} - 3i_x) + 3i_x$$

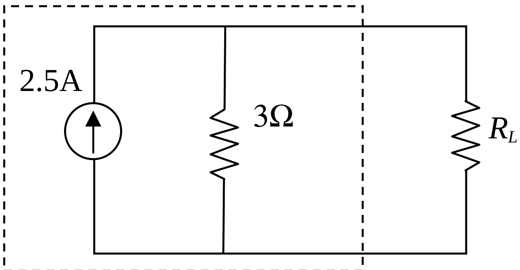
We have $i_x = \dfrac{V_{ex}}{15}$ and

$$V_{ex} = 6\left(I_{ex} - \frac{V_{ex}}{5}\right) + \frac{V_{ex}}{5} = 6I_{ex} - V_{ex}.$$

Hence, $R_N = \dfrac{V_{ex}}{I_{ex}} = 3\,\Omega$



The Norton's equivalent circuit is



2. (50%) What is the maximum power can be transferred to the load resistance R_L.

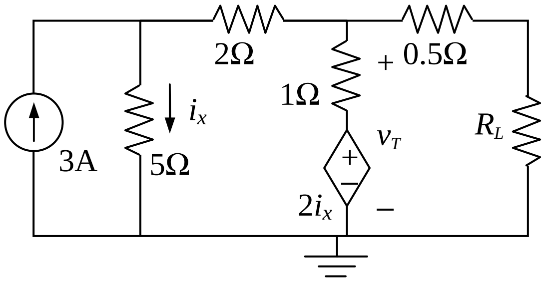
<u>Sol:</u>

(I) Take away R_L to find open voltage v_T

Since $(2+1)(3-i_x)+2i_x=5i_x$,

we have $i_x=1.5$ and

$v_T=(3-i_x)+2i_x=4.5$ V.

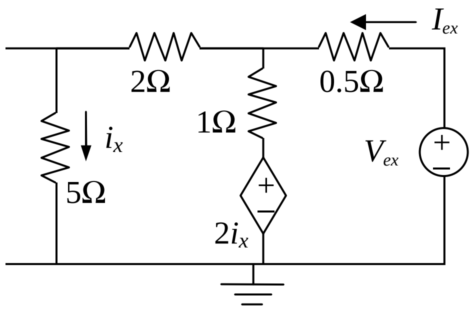


(II) Take away source and add an extra source

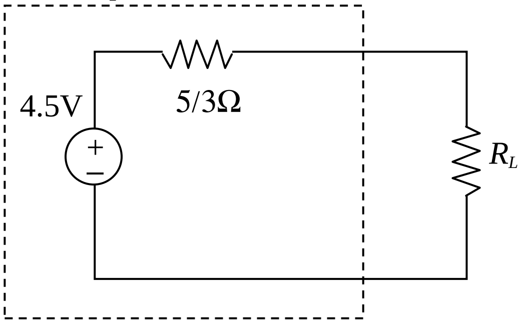
$V_{ex}=0.5I_{ex}+7i_x=0.5I_{ex}+(I_{ex}-i_x)+2i_x$

We have $i_x=0.1V_{ex}=I_{ex}/6$

and thus $R_T=\dfrac{V_{ex}}{I_{ex}}=\dfrac{5}{3}$.



The Thevenin's equivalent circuit is



Choose R_L=5/3 Ω, then the maximum power is $P_{max}=\dfrac{4.5}{2}\cdot\dfrac{4.5}{5/3+5/3}=3.0375\text{ W}$

NCTU Summer Course <Electrical Circuits> Quiz-07

________大學 ___________系 學號_____________ 姓名_________________

1. (50%) Consider a system described by the following ODE:

$$2\dot{y}(t)+3y(t)=12\,,\quad y(0.5)=1$$

Find $y(t)$, for $t\geq 0.5$. What is the time constant of this system?

Sol:

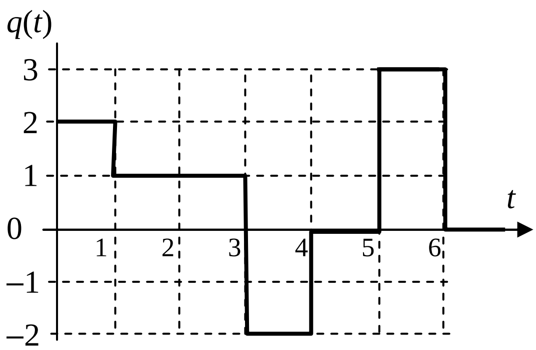
$$y(t)=-3e^{-1.5(t-0.5)}+4$$

Time constant is 0.667 sec.

2. (50%) A piecewise continuous function $q(t)$ is depicted below. Express $q(t)$ by the combination of step functions $u(t-\alpha)$ where α is an integer.

Sol:

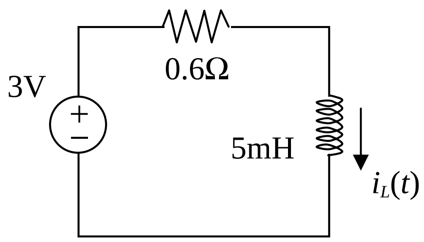
$$q(t)=2u(t)-u(t-1)-3u(t-3)$$
$$+2u(t-4)+3u(t-5)-3u(t-6)$$



NCTU Summer Course <Electrical Circuits> Quiz-08

________大學 ___________系 學號_____________ 姓名_________________

1. (50%) Consider the *RL* circuit shown below, where the initial inductor current is $i_L(0) = 1\,\text{A}$. What is the inductor current $i_L(t)$ for $t \geq 0$?



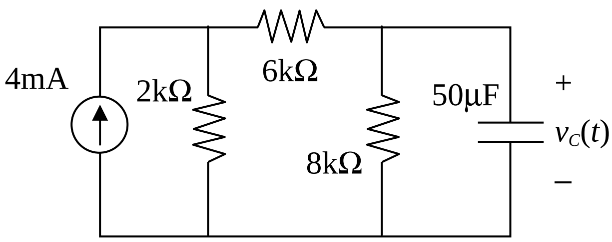
Sol:

R/L=0.6/5m=120

$i_L(0) = 1\,\text{A}$, $i_L(\infty) = 5\,\text{A}$

$i_L(t) = -4e^{-120t} + 5\,\text{A}$

2. (50%) Consider the circuit shown below, where the initial capacitor voltage is $v_C(0) = 1\,\text{V}$. What is the capcitor voltage $v_C(t)$ for $t \geq 0$?



Sol:

v_T=4V

R_T=4kΩ

$1/R_TC$=5

$v_C(0) = 1\,\text{V}$, $v_C(\infty) = 4\,\text{V}$

$v_C(t) = -3e^{-5t} + 4\,\text{V}$