1. (10%) Assume a current $i(t)$=10mA passes through a long electrical line, whose cross area is $a = 7\mu\text{m}^2$. Let the density be $n = 6.3\times10^{28}$ particles/m^3 and the charge of each particle be $q = 1.6\times10^{-19}\text{C}$. What is the average velocity of the particles moving along the electrical line?

Sol: Since $i = \dfrac{Q}{T} = \dfrac{Lanq}{L/v} = anqv$, we have the average velocity

$$v = \frac{i}{anq} = \frac{0.01}{7\times10^{-6}\times6.3\times10^{28}\times1.6\times10^{-19}} = \frac{1}{70.56\times10^{5}} = 1.42\times10^{-7}\,\text{m/sec}$$

2. (5%) In Fig-1, select a tree and show the cut surfaces S_i, i=1,2,...

Sol:

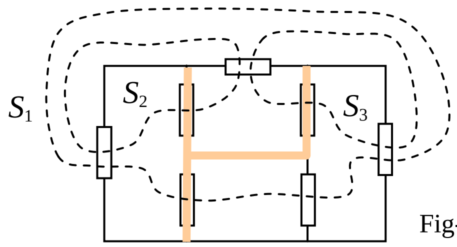


Fig-1

3. (15%) In Fig-2, find the equivalent resistance R_{eq}.

Sol:

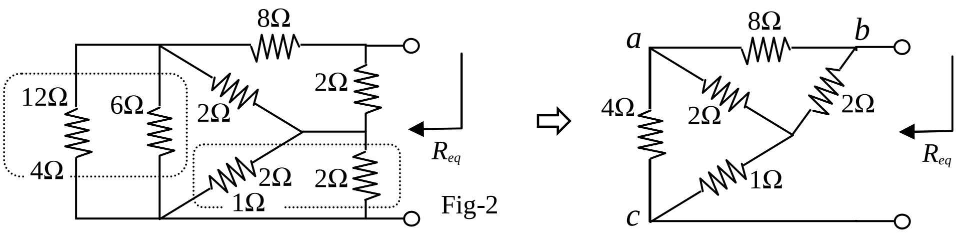
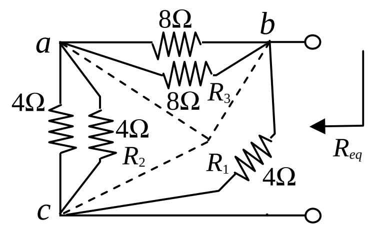


Fig-2

Y-Δ transform:

$$R_1R_a = R_2R_b = R_3R_c = R_aR_b + R_bR_c + R_cR_a$$
$$\Rightarrow 2R_1 = 2R_2 = R_3 = 4+2+2 = 8$$
$$\Rightarrow R_1 = 4,\ R_2 = 4,\ R_3 = 8$$



$$R_{eq} = 4/\!/\big((4/\!/4)+(8/\!/8)\big) = 4/\!/(2+4) = 4/\!/6 = 24/(4+6) = 2.4\,\Omega$$

4. (15%) In Fig-3, write the nodal voltage equation ***Fv***=***Gu***.

Sol:

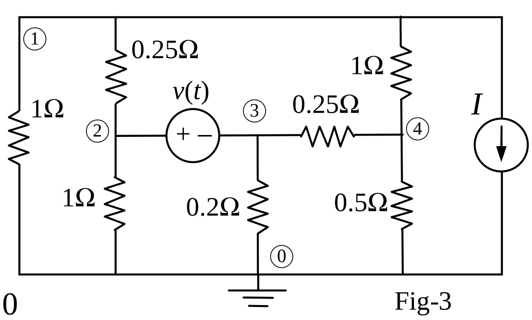


Fig-3

KCL①: $v_1 + 4(v_1 - v_2) + (v_1 - v_4) = -I$

$\Rightarrow 6v_1 - 4v_2 - v_4 = -I$

KVL②③: $v_2 - v_3 = v(t)$

KCL②③: $4(v_2 - v_1) + 4(v_3 - v_4) + v_2 + 5v_3 = 0$

$\Rightarrow -4v_1 + 5v_2 + 9v_3 - 4v_4 = 0$

KCL④: $(v_4 - v_1) + 4(v_4 - v_3) + 2v_4 = 0 \quad \Rightarrow -v_1 - 4v_3 + 7v_4 = 0$

The nodal voltage equation is

$$\underbrace{\begin{bmatrix} 6 & -4 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ -4 & 5 & 9 & -4 \\ -1 & 0 & -4 & 7 \end{bmatrix}}_{\boldsymbol{F}} \cdot \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}}_{\boldsymbol{v}} = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\boldsymbol{G}} \cdot \underbrace{\begin{bmatrix} I \\ v(t) \end{bmatrix}}_{\boldsymbol{u}}$$

5. (15%) In Fig-4, solve i through the resistor 2Ω based on the mesh analysis.

Sol:

KCL$\boxed{1}\boxed{2}$: $i_1 - i_2 = -3v_x = -3i_2 \quad \Rightarrow i_1 = -2i_2$

KVL$\boxed{1}\boxed{2}$: $2i_1 + i_2 + 2(i_2 - i_3) = 0$

$\Rightarrow 2i_1 + 3i_2 - 2i_3 = 0 \quad \Rightarrow i_2 = -2i_3$

KVL$\boxed{3}$: $2(i_3 - i_2) + i_3 = -7$

$\Rightarrow 3i_3 - 2i_2 = -7 \quad \Rightarrow 7i_3 = -7 \Rightarrow i_3 = -1$

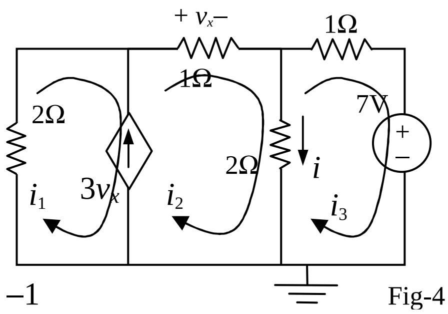
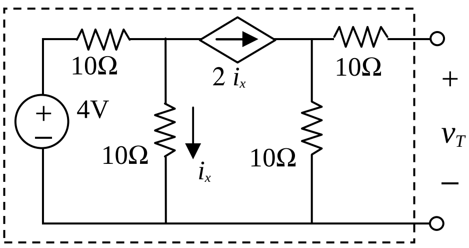


Fig-4

Hence, $i = i_2 - i_3 = -2i_3 - i_3 = -3i_3 = 3$ A

6. (15%) In Fig-5, write the Thevenin equivalent circuit through R_L. What is the resistance R_L that can receive the maximum power? What is the maximum power?

Sol:



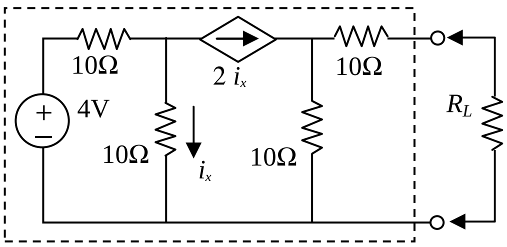
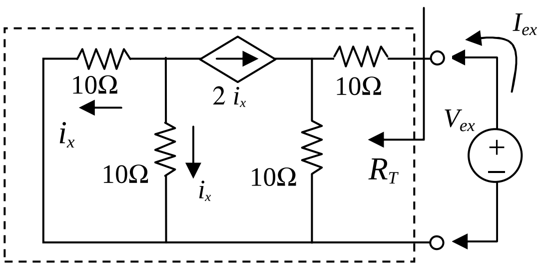


Fig-5

$4 = 10(i_x + 2i_x) + 10i_x \Rightarrow i_x = 0.1$

$v_T = 10 \times 2i_x = 2$ V



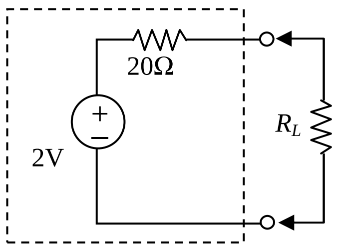
From the circuit on the right,

we have $i_x + i_x + 2i_x = 0 \Rightarrow i_x = 0$

Hence, $R_T = 10 + 10 = 20\,\Omega$

The Thevenin equivalent circuit is shown on the right.

If R_L=20Ω, it can receive the maximum power.



The voltage across R_L is $v_L = \dfrac{20}{20+20} \times 2 = 1\,\text{V}$

and the current is $i_L = \dfrac{2}{20+20} = 0.05$ A

Therefore, the maximum power is $p_{L\,max} = v_L i_L = 1 \times 0.05 = 0.05\text{W}$

7. (10%) In Fig-6, consider a voltage source $v(t)=2u(t)-3u(t-1)+2u(t-4)-u(t-5)$ and apply it to a resistor $R=4\Omega$. What is the average power dissipated by R for $0<t<6$?

<u>Sol:</u>

The curve of $v(t)$ is shown on the right.

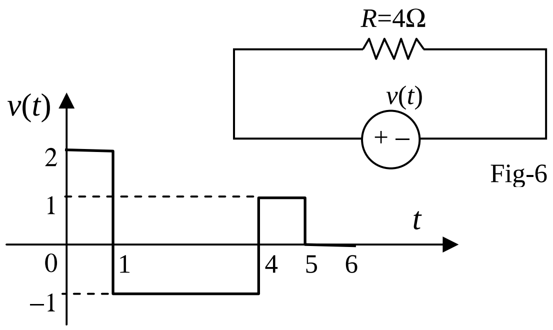


Fig-6

The power is $p(t)=\dfrac{v^2(t)}{R}$ and

the average power is

$$P_{av}=\frac{1}{6}\int_0^6 \frac{v^2(t)}{4}dt$$

$$=\frac{1}{6}\left(\int_0^1 \frac{2^2}{4}dt+\int_1^4 \frac{(-1)^2}{4}dt+\int_4^5 \frac{1^2}{4}dt\right)=\frac{1}{6}\left(1+\frac{3}{4}+\frac{1}{4}\right)=\frac{1}{3}\text{ W}$$

8. (15%) In Fig-7, a constant voltage source $v(t)=5$V is connected to a series RC circuit with $R=40\text{k}\Omega$ and $C=5\mu$F. If the initial voltage across the capacitor is $v_C(0)=1$V, then what is the current $i_R(t)$ for $t>0$?

Sol:

The system equation is

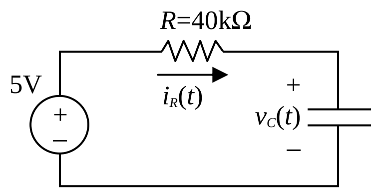


Fig-7

$$\dot{v}_C(t)=\frac{1}{C}i_C(t)=\frac{5-v_C(t)}{RC}=25-5v_C(t)$$

$$\Rightarrow \dot{v}_C(t)+5v_C(t)=25$$

Hence, $\Rightarrow v_C(t)=-4e^{-5t}+5$

We have $i_R(t)=\dfrac{5-v_C(t)}{40\times10^3}=0.1\times e^{-5t}$ mA