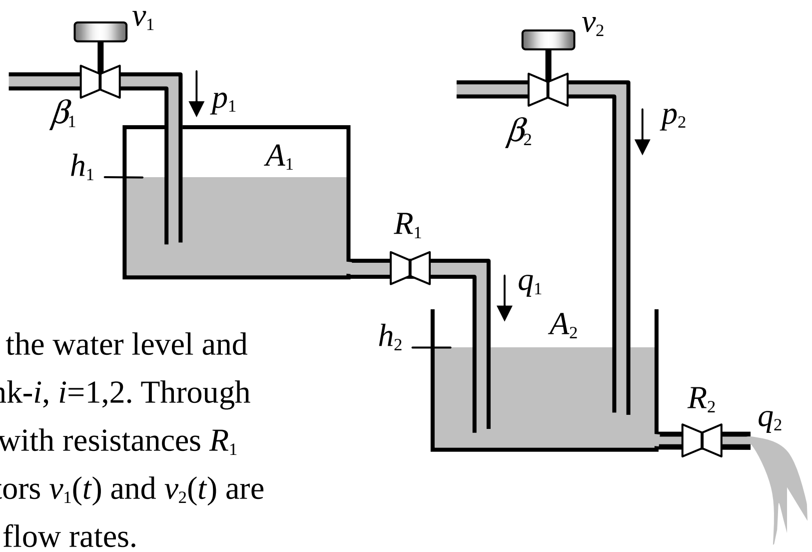
Final Exam of Dynamic System Analysis and Simulation, 2013/6/17 EF

1. Neglect all the physical units in this problem to design a two-tank water level controller. The system is depicted as below:



Let $h_i(t)$ and A_i be the water level and surface area of tank-i, i=1,2. Through two linear valves with resistances R_1 and R_2, two actuators $v_1(t)$ and $v_2(t)$ are used to adjust the flow rates.

Given A_1, A_2, R_1 and R_2, the dynamic equation can be written as

$$\dot{h}_1(t) = -2h_1(t) + v_1(t)$$
$$\dot{h}_2(t) = 2h_1(t) - h_2(t) + v_2(t)$$

In order to maintain fixed water levels h_{1d} and h_{2d}, we tune the actuators as

$$v_1(t) = 2h_{1d} + u(t)$$
$$v_2(t) = -2h_{1d} + h_{2d}$$

By setting state variables $x_1(t) = h_1(t) - h_{1d}$ and $x_2(t) = h_2(t) - h_{2d}$, we have the state equation described as beow:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t) \qquad (1)$$

where $\boldsymbol{x}(t) = [x_1(t) \quad x_2(t)]^T$.

(A) Determine the system matrices $\boldsymbol{A}$ and $\boldsymbol{b}$, and verify the system controllability.

(B) If the state variables $x_1(t)$ and $x_2(t)$ are measurable, please design a state feedback control $u(t)=-\boldsymbol{k}\boldsymbol{x}$ by assigning –1 and –4 as the eigenvalues of the controlled system.

(C) If only $x_2(t)$ is measurable, i.e., $y(t)=x_2(t)$, please verify the system observability.

(D) Design a Luenberger observer for the case in (C) by choosing –3+j and –3–j as the convergent rates of the estimation error $\tilde{\boldsymbol{x}}(t) = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)$, where $\hat{\boldsymbol{x}}(t)$ is the estimated state of the observer.

(E) Consider the system $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t)$ given in (1) with initial condition $x_1(0)=1$ and $x_2(0)=2$. Simulate the case in (C) and (D) by using the control input $u(t)=-\boldsymbol{k}\hat{\boldsymbol{x}}(t)$ where the control gain vector $\boldsymbol{k}$ is obtained in (B). Show your results of $x_1(t)$, $x_2(t)$, $\tilde{x}_1(t)$ $\tilde{x}_2(t)$ and $u(t)$ for $t \in [0,6]$.

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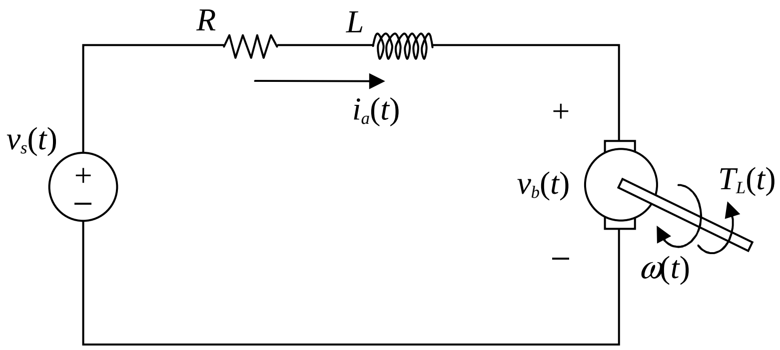
2. Neglect the physical units in this problem to design an angular position control of the DC motor depicted as below:



Its dynamic equation can be expressed as

$$Ri_a(t) + L\frac{di_a(t)}{dt} + k\omega(t) = v_s(t)$$

$$J\frac{d\omega(t)}{dt}(t) + B\omega(t) - ki_a(t) = -T_L(t)$$

$$\omega(t) = \frac{d\theta(t)}{dt}$$

where R=10, L=0.03, J=0.005, k= 0.01 and B= 0.001. Let the armature current $i_a(t)$, angular velocity $\omega(t)$ and angular position $\theta(t)$ be measurable. Assume the motor is coupled with a rotary payload with inertia J_L=0.002, i.e., $T_L(t) = J_L\frac{d\omega(t)}{dt}$.

(F) Choose the input $u(t)=v_s(t)$ and the state variables as $x_1(t) = i_a(t)$, $x_2(t) = \omega(t)$ and $x_3(t) = \theta(t) - \theta_d$, where θ_d is the desired angular position. If the state equation is $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t)$, then what are $\boldsymbol{A}$ and $\boldsymbol{b}$. Verify that the system is controllable.

(G) With the use of eigenvalues –10, –30+j10, –30–j10, design a state-feedback control $u(t)=-\boldsymbol{kx}(t)$ such that the angular position $\theta(t)$ converges to θ_d as $t\to\infty$.

(H) Simulate the controlled system with $\theta(0)$=0 and θ_d=1. Show your results of $i_a(t)$, $\omega(t)$, $\theta(t)$ and $u(t)$ for $t \in [0,4]$.

(I) Let the desired angular positrion be a sinusoidal function, for example $\theta_d(t)=sin(5t)$, and the state-feedback control $u(t)=-\boldsymbol{kx}(t)$ in (G) is used. What will happen?
[Hint: You can simulate the system first for $t \in [0,10]$ and then tell what happens from the difference between $\theta(t)$ and $\theta_d(t)=sin(5t)$.]